Math 2270-004
Homework due February 7.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; only the underlined problems need to be handed in. The Wednesday quiz will be drawn from all of these concepts and from these or related problems.

2.1 Matrix operations, including matrix multiplication
1, **3**, **9**, **11**, **23**, **25**, **27**

2.2 matrix inverses
1, 3, 5, **7, 13 19, 21, 25,** 31

2.3 characterizations of invertible matrices
5, **11,** 13, **15, 17, 19,** 21, **33,** 35

w4.1 Let

$$A := \begin{bmatrix} 5 & 2 \\ 7 & 1 \end{bmatrix}$$

You will be working with this matrix in various ways, in each of a-f below.

w4.1a) Find A^{-1} using the special (adjoint) formula for inverses of 2 by 2 matrices on page 105.

w4.1b) Find A^{-1} using the Gaussian elimination algorithm, where you reduce A augmented with the identity matrix. (Which do you prefer in this case, the method in a or the method in b?)

w4.1c) Use your formula for A^{-1} to solve the system

$$\begin{bmatrix} 5 & 2 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}.$$

Check that your solution is correct by verifying that it makes the original equation true.

w4.1d) Use your formula for A^{-1} to solve the system

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 7 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \end{bmatrix}.$$

Check that your solution is correct by verifying that it makes the original equation true.

w4.1e) Use A^{-1} to solve for the mystery matrix X in the following matrix equation. Check that your answer works!

$$X \begin{bmatrix} 5 & 2 \\ 7 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 2 & -1 \\ 3 & 3 \end{bmatrix}.$$

w4.1f) Use matrix algebra to solve for X. Verify that your answer works (with technology or by hand)! Hint: in order to factor out the matrix X (on the left), on the left side of the equation below, rewrite $-3X$ as

$-3\,I\,X$, where I is the 2×2 identity matrix.

$$\begin{bmatrix} 8 & 2 \\ 7 & 4 \end{bmatrix} X - 3\,X = \begin{bmatrix} 1 & 2 \\ 5 & 1 \end{bmatrix}.$$

Gaussian elimination algorithm to deduce whether inverses exist, and to find them when they do:
w4.2) Use the Gaussian elimination algorithm to determine that the matrix A below is not invertible, whereas the matrix B is. Use the algorithm that begins by augmenting a matrix with the identity matrix, in order to find the inverse matrix B^{-1}.

$$A := \begin{bmatrix} -1 & 1 & -4 \\ -1 & -1 & 2 \\ 4 & 1 & 1 \end{bmatrix} \qquad B := \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 3 \\ -2 & 1 & -2 \end{bmatrix}$$