Math 2270-004
Week 14-15 homework,
due Tuesday April 24 at 6:00 p.m.

7.1-7.2 Diagonalization of symmetric matrices and quadratic forms

7.2: 1, **3**, **5**, **7**, **9**, **11**, **16** (use technology on 16), **21**

w14.1 Find a spectral decomposition for the symmetric matrix and orthonormal eigenbasis you used in problem 7 above. (See page 400 or class notes).

w14.2 For the two equations below
(i) Diagonalize the quadratic form on the left of each equation. Classify the conic section.
(ii) Pick your orthonormal eigenbasis $B = \{\underline{u}_1, \underline{u}_2\}$ so that it is positively oriented. Following the text, write $[\underline{x}]_B = \underline{y}$. So, $P = [\underline{u}_1, \underline{u}_2]$ is the change of coordinates matrix, $P_{E \leftarrow B}$. Sketch the conic using the rotated coordinate system.

a) $x_1 x_2 = 2$
b) $6x_1^2 + 4x_1x_2 + 3x_2^2 = 1.$

w14.3 Outer product.
a) Compute

$$\begin{bmatrix} 1 & 2 \\ -2 & 4 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

using the outer product method.
b) Using the dot product formulation for $A_{m \times p} B_{p \times n}$, we know

$$entry_{kl} A\,B = row_k(A) \cdot col_l(B) = \sum_{j=1}^{p} a_{kj}\, b_{jl}.$$

Explain why this is the same value you get for the $k\,l-$ entry of the outer sum expression

$$\sum_{j=1}^{p} \underline{a}_j\, \underline{b}_j$$

where the $\{\underline{a}_j\}$ are the columns of A and the $\{\underline{b}_j\}$ are the rows of B.

$$A = \begin{bmatrix} | & | & & | \\ \underline{a}_1 & \underline{a}_2 & \cdots & \underline{a}_p \\ | & | & & | \\ | & | & & | \end{bmatrix} \qquad B = \begin{bmatrix} ---\underline{b}_1--- \\ ---\underline{b}_2--- \\ \vdots \\ ---\underline{b}_p--- \end{bmatrix}.$$