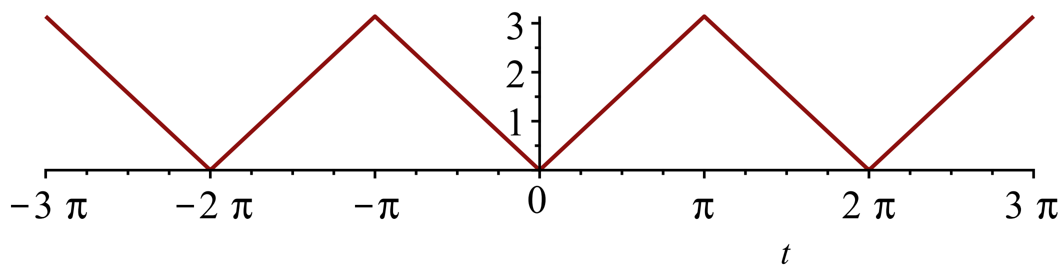
Math 2270-004
Week 13-14 homework,
due April 18.

6.6 Linear models for data fitting
1, **7**, and exercise **w13.1** below, in the height-weight scaling law notes.

6.7-6.8 Inner product spaces
6.7.25 (Legendre polynomials): For functions in $C[-1, 1]$ Use Gram-Schmidt to find an orthonormal basis for $W = span\{1, t, t^2\}$, with respect to the inner product

$$\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)\, dx.$$

w13.2a) Find the Fourier series expansion of the 2π-periodic "tent function", which on the interval $[-\pi, \pi]$ is given by $f(t) = |t|$.



Hint: You may use the fact that for any even function $g(t)$ (i.e. $g(-t) = g(t)$),

$$\int_{-a}^{a} g(t)\, dt = 2\int_{0}^{a} g(t)\, dt.$$

And the fact that for any odd function $h(t)$ (i.e. $h(-t) = -h(t)$),

$$\int_{-a}^{a} h(t)\, dt = 0.$$

This will result in your particular Fourier expansion being just a *cosine* series, and you'll be able to compute the cosine coefficients using integrals from 0 to π.

w13.2b) Plot the graph of the sum of the first 6 non-zero terms in the Fourier expansion for the tent function, on the interval $[-3\pi, 3\pi]$ using technology, and hand-in a printout.

7.1 Diagonalization of symmetric matrices
1, **2**, **3**, **4**, **5**, **6**, **7**, **9**, **11**, **15**, **17**, **19**, 23, **25**, 31, 33 Note: the text gives you the eigenvalues for the matrices in 17, 19.

Math 2270-004 Spring 2018

A Power Law For Human Heights and Weights

Body Mass Index

A person's BMI is computed by dividing their weight by the square of their height, and then multiplying by a universal constant. If you measure weight in kilograms, and height in meters, this constant is the number one. If you measure height in inches and weight in pounds then the formula is

$$BMI = 703\frac{w}{h^2}$$

The graph of heights and weights for which BMI has a constant value B is the parabola

$$w = \frac{B}{703} \cdot h^2.$$

Thus, the assumption underlying BMI is that for adults at equal risk levels (but different heights), weight should be proportional to the square of height. This is a historical accident and at some point became a dogma. The BMI was popularized in the 1960's in the U.S., by proponents who were initially unware that they were repeating history. It is easy to deduce that if people were to scale equally in all directions when they grew, weight would scale as the cube of height. That particular power law seems a little high, since adults don't look like uniformly expanded versions of babies; we seem to get relatively stretched out length-wise when we grow taller. One would expect the best predictive power to be somewhere between 2 and 3. If the power is much larger than 2 then one could argue that the body mass index might need to be modified to reflect this fact.

It turns out a Belgian demographer, Adolphe Quetelet, also called the "Father of Statistics", originally proposed a power of p=2 for adults, based on his own data analysis during the early 1800's. In a footnote which history has forgotten, he said that a power of 2.5 is more appropriate if you want an approximation for people of all ages. He actually wrote that the square of the weight should scale like the fifth power of the height, because pre-calculators, fractional powers were harder for people to deal with. My recollection is that this footnote appears in the 1835 publication "Sur l'homme et le développement de ses facultés, ou Essai de physique sociale". I have read the footnote.

There is (or at least there was, 20 years ago) a database at the U.S. Center for Disease Control, of national body data collected between 1976 and 1980. From this data I have extracted the median heights and weights for boys and girls, age 2-19. The national data is shown below; heights are given in inches and weights are in pounds.

w13.1) Find the power law

$$w = C\,h^p$$

predicted by this data, by finding a least squares line fit to the ln-ln data.

```
> with(LinearAlgebra) :
> A1 := Matrix([[35.9, 38.9, 41.9, 44.3, 47.2, 49.6, 51.4, 53.6, 55.7,
        57.3, 59.8, 62.8, 66.0, 67.3, 68.4, 68.9, 69.6, 69.6],
       [29.8, 34.1, 38.8, 42.8, 48.6, 54.8, 60.8, 66.5, 76.8, 82.3, 93.8,
       106.8, 124.3, 132.6, 142.1, 145.1, 155.3, 153.2]]) :
```

boys height-weight data ages 2-18

35.9	38.9	41.9	44.3	47.2	49.6	51.4	53.6	55.7	57.3	59.8	62.8	66.0	67.3	68.4	68.9	69.6	69.6
29.8	34.1	38.8	42.8	48.6	54.8	60.8	66.5	76.8	82.3	93.8	106.8	124.3	132.6	142.1	145.1	155.3	153.2

```
> A2 := Matrix([[35.4, 38.4, 41.1, 43.9, 46.6, 48.9, 51.4, 53.1, 55.7,
       58.2, 61.0, 62.6, 63.3, 64.2, 64.3, 64.2, 64.1, 64.5],
      [28.0, 32.6, 36.8, 41.8, 47.0, 52.5, 60.8, 65.5, 76.1, 89.0, 100.1,
      108.1, 117.1, 117.6, 122.6, 128.8, 124.5, 126.0]]) :
```

girl's data:

35.4	38.4	41.1	43.9	46.6	48.9	51.4	53.1	55.7	58.2	61.0	62.6	63.3	64.2	64.3	64.2	64.1	64.5
28.0	32.6	36.8	41.8	47.0	52.5	60.8	65.5	76.1	89.0	100.1	108.1	117.1	117.6	122.6	128.8	124.5	126.0

age	*boy height*	*weight*	*girl height*	*weight*
2	35.9	29.8	35.4	28.0
3	38.9	34.1	38.4	32.6
4	41.9	38.8	41.1	36.8
5	44.3	42.8	43.9	41.8
6	47.2	48.6	46.6	47.0
7	49.6	54.8	48.9	52.5
8	51.4	60.8	51.4	60.8
9	53.6	66.5	53.1	65.5
10	55.7	76.8	55.7	76.1
11	57.3	82.3	58.2	89.0
12	59.8	93.8	61.0	100.1
13	62.8	106.8	62.6	108.1
14	66.0	124.3	63.3	117.1
15	67.3	132.6	64.2	117.6
16	68.4	142.1	64.3	122.6
17	68.9	145.1	64.2	128.8
18	69.6	155.3	64.1	124.5
19	69.6	153.2	64.5	126.0

A graph of the best line fit to the national ln − ln data. It's a pretty good fit! (Infants are a little heavier than the line predicts, adolescent data is slightly below the line, and as adults mature they rise a bit above the line.