## Wed Sept 5

• 1.7 Linear dependence/independence continued.

Announcements: "quiz today

· usually you'll get back all assignments within a week

(or sooner). How delayed this week

· HW3 is mostly posted. All by tonight

Warm-up Exercise: Exercise 1 in today's notes

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n = \vec{0}$$
 has non-zero solutions  $\vec{c}$ 

Note that the set of vectors  $\{\underline{v}_1,\underline{v}_2, \dots \underline{v}_n\}$  is <u>linearly dependent</u> if and only if there are non-zero solutions c to the homogeneous matrix equation

$$A \underline{c} = \underline{0}$$

for the matrix  $A = \begin{bmatrix} \underline{\mathbf{y}}_1 & \underline{\mathbf{y}}_2 & \dots & \underline{\mathbf{y}}_n \end{bmatrix}$  having the given vectors as columns. Thus all linear independence/dependence questions can be answered using reduced row echelon form and facts about homogeneous matrix solutions.

Exercise 1) Show that the vectors

$$\underline{\boldsymbol{v}}_{1} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \ \underline{\boldsymbol{v}}_{2} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \underline{\boldsymbol{v}}_{3} = \begin{bmatrix} -1 \\ 6 \\ 4 \end{bmatrix}$$

are linearly dependent (even though no two of them are scalar multiples of each other). What does this mean geometrically about the span of these three vectors?

to solve
$$c_{1} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_{2} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + c_{3} \begin{bmatrix} -1 \\ 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
are there sol has besides  $c_{1} = c_{2} = c_{3} = 0$ ?

Hint: You might find this computation useful:

## Exercise 2) Are the vectors

$$\underline{\boldsymbol{v}}_{1} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \ \underline{\boldsymbol{v}}_{2} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \ \underline{\boldsymbol{w}}_{3} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

linearly independent or dependent? Hint:

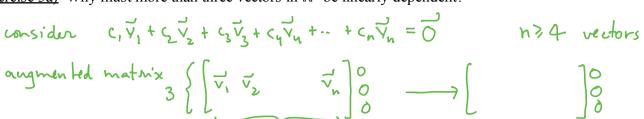
$$\begin{bmatrix}
1 & -1 & 1 \\
0 & 2 & 1 \\
2 & 0 & 1
\end{bmatrix}
0$$

$$\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
0$$
Vector eqtn
$$\begin{bmatrix}
c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3 = \vec{0}
\end{bmatrix}$$

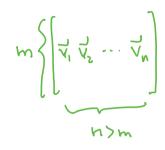
$$\begin{bmatrix}
c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3 = \vec{0}
\end{bmatrix}$$

$$\begin{bmatrix}
c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3 = \vec{0}
\end{bmatrix}$$
Independent

Exercise 3a) Why must more than three vectors in  $\mathbb{R}^3$  be linearly dependent?



3b) How about more than m vectors in  $\mathbb{R}^m$ ?

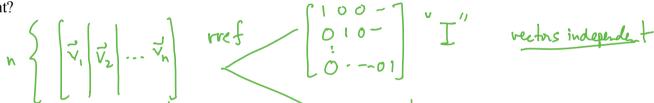


at most 3 pivots at least n-3 free so, lots of dependencies.

at most in pivots at clast inm free variables So, lots of dependencies

<u>3c</u>) If you are given a set of exactly n vectors in  $\mathbb{R}^n$  how can you check whether or not they are linearly independent?





<u>3e</u>) If  $\{\underline{v}_1,\underline{v}_2, \dots \underline{v}_n\}$  is a set of exactly *n* vectors in  $\mathbb{R}^n$  what condition on the reduced row echelon form of  $A = [\underline{v}_1, \underline{v}_2, \dots \underline{v}_n]$  guarantees and is required so that the vectors span  $\mathbb{R}^n$ ? Compare with  $\underline{3c}$ .