Exercise 3 Illustrate the linear transformation theorem with the projection function $T:\mathbb{R}^3 \to \mathbb{R}^2$, by writing T as a matrix transformation.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

formula (dot product way)

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

theorem way

$$A = \begin{bmatrix} T(\vec{e}_1) & T(\vec{e}_2) & T(\vec{e}_3) \end{bmatrix} = \begin{bmatrix} T\begin{bmatrix}1\\0\\0\end{bmatrix} & T\begin{bmatrix}0\\1\\0\end{bmatrix} & T\begin{bmatrix}0\\0\\1\end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

same

is this onto? Yes.

is this 1-1? No e.g. $T\begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$, also $T\begin{bmatrix}1\\2\\27\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$.

check: this agrees with general conditions on rref(A)

<u>Exercise 4</u> Find a matrix formula for the linear transformation $\boxed{T:\mathbb{R}^2 \to \mathbb{R}^3}$ which satisfies

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=\begin{bmatrix}0\\2\\-1\end{bmatrix} \quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=\begin{bmatrix}-2\\0\\0\end{bmatrix}.$$

~~And sketch the mapping diagram.~~

$$T(\vec{x}) = \begin{bmatrix}0 & -2\\ 2 & 0\\ -1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

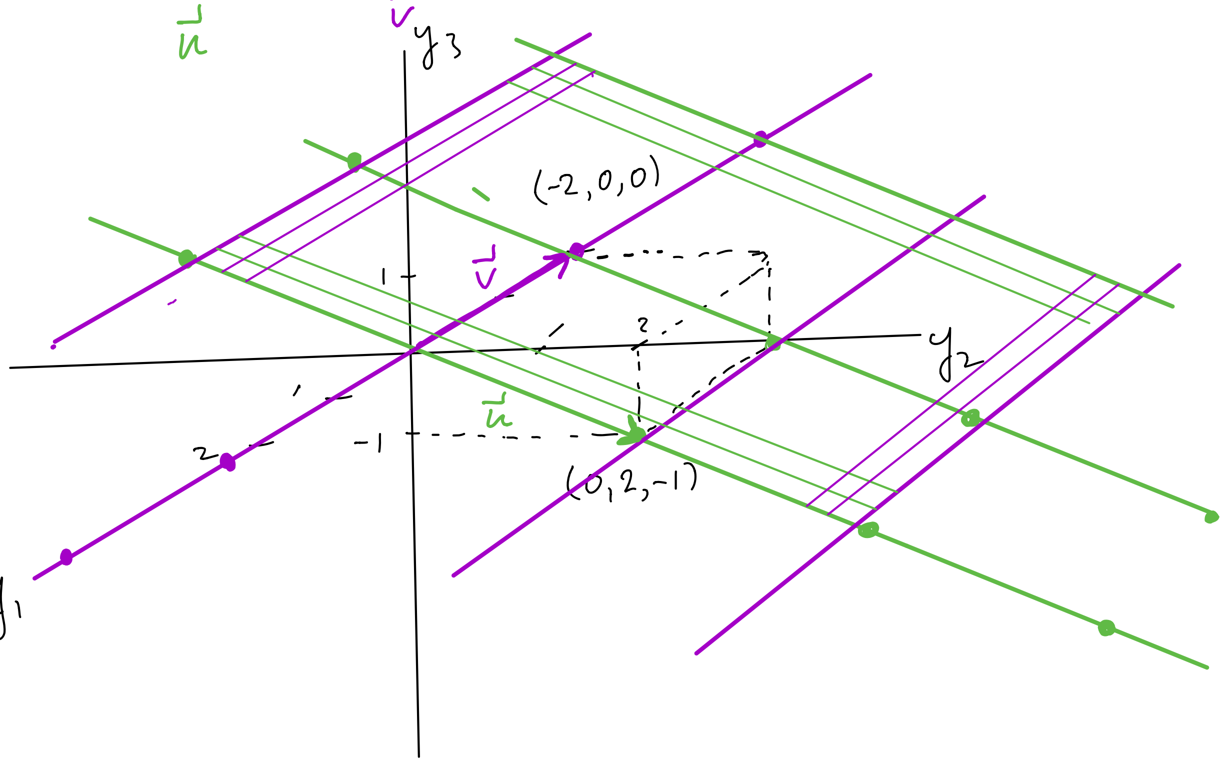
$$A = \begin{bmatrix} T(\vec{e}_1) & T(\vec{e}_2)\end{bmatrix} = \begin{bmatrix}0 & -2\\ 2 & 0\\ -1 & 0\end{bmatrix}$$

<u>onto?</u> <u>NO</u>:

<u>1-1?</u> <u>yes</u>

range is a plane in $\mathbb{R}^3$

$$\left\{ x_1 \underbrace{\begin{bmatrix}0\\2\\-1\end{bmatrix}}_{\vec{u}} + x_2 \underbrace{\begin{bmatrix}-2\\0\\0\end{bmatrix}}_{\vec{v}} \text{ such that } x_1, x_2 \in \mathbb{R}\right\}.$$



When you learned about functions in a previous course, the following were key ideas:

Definitions: The function $f: X \to Y$ is *one to one* (sometimes called *injective*) if each image point of f arises from exactly one input value. In other words,

solutions to $f(x) = b$ are unique

$$f(x_1) = f(x_2) \;\Rightarrow\; x_1 = x_2.$$

The function $f: X \to Y$ is *onto* (sometimes called *surjective*) if for each $y \in Y$ there is at least one $x \in X$ so that $f(x) = y$.

solutions to $f(x) = b$ exist for each $b \in Y$

If $f: X \to Y$ is a function, then the function $g: Y \to X$ is called an (the) *inverse function* for f if and only if

$$g(f(x)) = x \quad \text{for all } x \in X \qquad \text{and} \quad f(g(y)) = y \quad \text{for all } y \in Y.$$

Theorem A function $f: X \to Y$ has an inverse function $g = f^{-1}$ if and only f is one to one and onto.

w

Exercise 5 For a linear function $T: \mathbb{R}^n \to \mathbb{R}^m$ with matrix A i.e. $T(\underline{\mathbf{x}}) = A\,\underline{\mathbf{x}}$

5a What is the pivot condition on the matrix A that makes the linear function T one to one?

when are sol'ns to $A\vec{x} = \vec{b}$ unique? (if they exist) : each col of A is a pivot column.

5b What is the pivot condition on the matrix A that makes the function T onto?

when can we always solve $A\vec{x} = \vec{b}$, i.e. for all $\vec{b}$

every row of rref(A) has a pivot

5c Is the transformation in Exercise 3 one to one? Is it onto?

5d Is the transformation in Exercise 4 one to one? is it onto?

Wed Sept 12

- 2.1 Matrix operations

Announcements: 2.1 HW: 1, 3, 5, 9, 11

(from today's class topics)

- quiz today
- we talked about the function concepts of 1-1 & onto (for Matrix transformations), Tues. notes

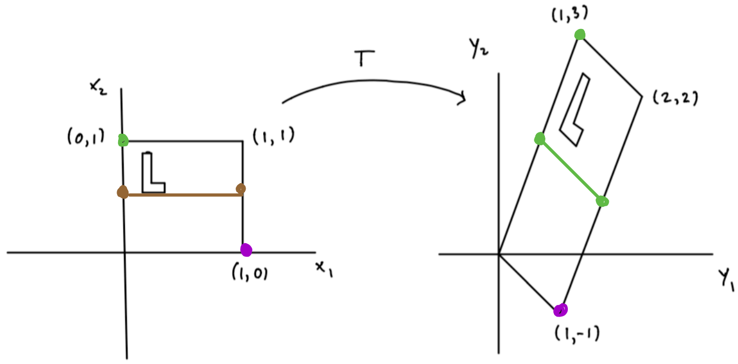
Warm-up Exercise: check with your neighbors to make sure you can do the last exercise in the HW you're handing in today.

Warmup: check with people around you to make sure you understand the last question in today's HW.

w3.5

a Use the transformation picture for a mystery linear transformation $T(\underline{x}) = A\,\underline{x}$ to reconstruct the matrix A.

$$A = \begin{bmatrix} T(\vec{e}_1) & T(\vec{e}_2) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$$



b) Sketch the line segment in the domain (above) whose position vectors are given parametrically by

$$L := \left\{ \begin{bmatrix} 0 \\ .5 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ 0 \le t \le 1 \right\}.$$

connects (0, .5) to (1, .5)

Then sketch the image line segment, $T(L)$, in the codomain (above) and provide a parametric formula for the image position vectors. $\vec{r}(t)$

$$A\begin{bmatrix} t \\ .5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} t \\ .5 \end{bmatrix} = t\begin{bmatrix} 1 \\ -1 \end{bmatrix} + .5\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\left(\text{or } A\left(\begin{bmatrix} 0 \\ .5 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = A\begin{bmatrix} 0 \\ .5 \end{bmatrix} + A\,t\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ .5 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} t \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} .5 \\ 1.5 \end{bmatrix} + t\begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$