

Math 2270-002 Week 11 notes

We will not necessarily finish the material from a given day's notes on that day. We may also add or subtract some material as the week progresses, but these notes represent an in-depth outline of what we plan to cover. These notes cover material in 5.4-5.5, 6.1.

Mon Nov 5

- 5.4-5.5 Brief intro to Matlab, continued discussion of change of variables to understand linear transformations, and introduction to complex eigenvalues and eigenvectors

Announcements:

Warm-up Exercise: Find eigenvalues & eigenspace bases for

$$A = \begin{bmatrix} 4 & 4 \\ -1 & 0 \end{bmatrix}$$

ans: $p(\lambda) = (\lambda - 2)^2$
 $E_{\lambda=2} = \text{span} \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$
use later.

So far, all of our eigendata has been real, but it's possible to have complex eigenvalues and eigenvectors as well, in which case we switch from considering real number scalars to considering complex number scalars and proceed as before. We'll introduce those ideas today and tomorrow, in section 5.5.

On Wednesday, I tentatively plan to begin Chapter 6 on "Orthogonality". This Chapter will lead to amazing applications, but it begins with a review and extension of dot product ideas related to angles between vectors and orthogonality in \mathbb{R}^n , that you learned about in multivariable Calculus for \mathbb{R}^2 and \mathbb{R}^3 . There will be a relatively short homework assignment due next Wednesday, containing preliminary Chapter 6 material.

On Friday we have our second midterm exam, which will cover sections 4.1-4.6, 4.9 (and google page rank), 5.1-5.5. I'll reserve a room for a problem session on Thursday, where we will go over a practice exam.

For your extra credit homework problem, and because you want to familiarize yourself with Matlab for the upcoming chapter as well, I'll demo Matlab.

Here are the two Matlab scripts I've created for this week. You can download these from either our public homework page or CANVAS. Put them into a directory and open them from Matlab. When you run the script with the big arrow at the top of your Matlab window all the commands will be executed. You can modify the script as desired.

The file "[matrixcomputations.m](#)" (The .m signifies matlab file.) :

```
% some matlab commands for matrices...if you want more just use google.
```

```
% enter a matrix row by row, each row terminates with ";"
```

```
A=[1 2 3; 4 5 6] %use spaces or commas to separate row entries
```

```
AA=[3 5 9;2 4 6]; %if you put a semi-colon at the end the computation  
%isn't echoed in the command window.
```

```
%rref computes reduced row echelon form. I named the result
```

```
% with an equals sign and a name, so
```

```
%that it would be saved to my workspace after running the script
```

```
B=rref(A)
```

```
% augmenting a matrix, either by rows or columns: first by row
```

```
b=[1,0,0] %row vector ..
```

```
C=[A;b] % add a 3rd row to A, using b
```

```
% use semicolons to separate rows
```

```
D=rref(C) % the 3 by 3 matrix D is invertible, since D is the identity
```

```
E=C^(-1) % inverse matrix...everything is decimals
```

```
F= C*E % this is how you multiply matrices! should get identity
```

```
% augmenting a matrix by columns
```

```
g=[-1;3] % a column vector with two entries
```

```
H=[A,g] % augment A with the column g
```

```
K=rref(H) %reduce the augmented matrix to find solutions to Ax=g
```

```
% eigenvalues and eigenvectors
```

```
L=[2,1,0;0,2,0;0,0,3]
```

```
lambda=eig(L) % eigenvalues
```

```
[M,N]=eig(L) % eigenvalues in first diagonal matrix,
```

```
%eigenvectors in second matrix
```

```
% note that this matrix is not diagonalizable
```

The file "PAC_team_rankings.m"

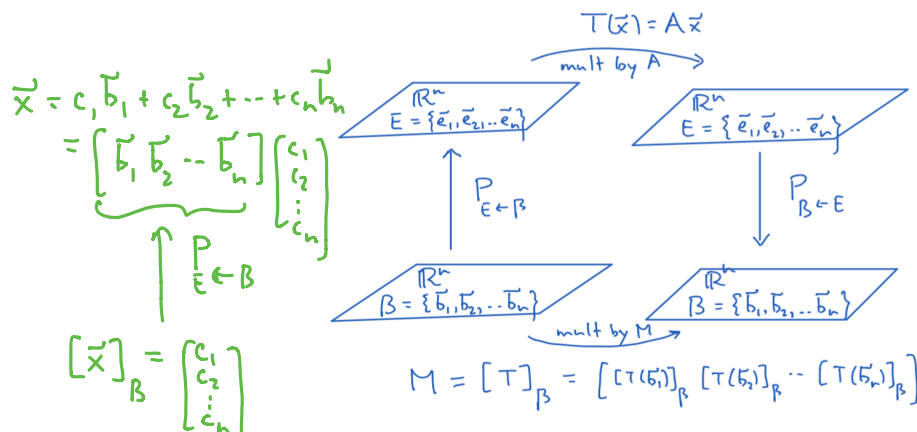
```
% PAC 12 Stochastic Matrix before past weekend
%  AZ  AZS  CAL  CU  OR  OSU  STA  UCLA  USC  UT  WA  WAS
SMa=[ 0, 0, 1/3, 0, 1/3, .25, 0, 0, 0, 0, 0, 0; %AZ
      0, 0, 0, 0, 0, .25, 0, 0, 1/3, 0, 0, 0; %AZS
      0, 0, 0, 0, 0, .25, 0, 0, 0, 0, .5, 0; %CAL
      0, 1/3, 0, 0, 0, 0, 0, 1/3, 0, 0, 0, 0; %CU
      0, 0, 1/3, 0, 0, 0, 0, 0, 0, 0, .5, 0; %OR
      0, 0, 0, 1/3, 0, 0, 0, 0, 0, 0, 0, 0; %OSU
      0, 1/3, 0, 0, 1/3, 0, 0, 0, 1/3, 0, 0, 0; %STA
      1/3, 0, 1/3, 0, 0, 0, 0, 0, 0, 0, 0, 0; %UCLA
      1/3, 0, 0, 1/3, 0, 0, 0, 0, 0, 0, 0, 1; %USC
      1/3, 0, 0, 0, 0, 0, .5, 1/3, 1/3, 0, 0, 0; %UT
      0, 1/3, 0, 1/3, 0, 0, 0, 1/3, 0, .5, 0, 0; %WA
      0, 0, 0, 0, 1/3, .25, .5, 0, 0, .5, 0, 0;] %WAS

%create a regular stochastic matrix with google fudge factor:
SMb=.15/12*ones(12,12)+.85*SMa

% the columns of a large power of SMb should be essentially identical,
% and are the equilibrium solution. The entries can be used to rank
% the Pac 12 teams, based on current records.
SMb^20
```

Returning to 5.4-5.5:

$$[T]_{\beta} = P_{\beta \leftarrow E} [T]_E P_{E \leftarrow \beta}$$



Exercise 1 (To review change of basis) Try to pick a better basis to understand the matrix transformation $T(\underline{x}) = A \underline{x}$, even though the matrix A is not diagonalizable. Compute $M = P^{-1}AP$ or compute M directly, to see if it really is a "better" matrix.

you could put almost anything there

better basis

$$A = \begin{bmatrix} 4 & 4 \\ -1 & 0 \end{bmatrix}$$

$$B = \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} = \{\vec{b}_1, \vec{b}_2\}$$

$$E_{\lambda=2} = \text{span} \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$$

check: $A \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} \checkmark$

$$M = [T]_B = \begin{bmatrix} [T(\vec{b}_1)]_B & [T(\vec{b}_2)]_B \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix}$$

$$T(\vec{b}_1) = 2\vec{b}_1 + 0\vec{b}_2$$

$$[T(\vec{b}_1)]_B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

So $M = \begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix} = 2 \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$T(\vec{b}_2) = \begin{bmatrix} 4 & 4 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

matrix of a horiz shear of strength -1

$$\begin{bmatrix} 4 \\ 0 \end{bmatrix}_B : \begin{bmatrix} 4 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

check, also

$$M = P^{-1}AP$$

$$= \begin{bmatrix} -2 & 0 \\ 1 & 1 \end{bmatrix}^{-1} A \begin{bmatrix} -2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\begin{array}{cc|c} -2 & 0 & 4 \\ 1 & 1 & 0 \\ \hline 1 & 0 & -2 \\ 1 & 1 & 0 \end{array} \quad \begin{matrix} c_1 = -2 \\ c_2 = 2 \end{matrix}$$

$$\begin{bmatrix} 4 \\ 0 \end{bmatrix}_B = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$