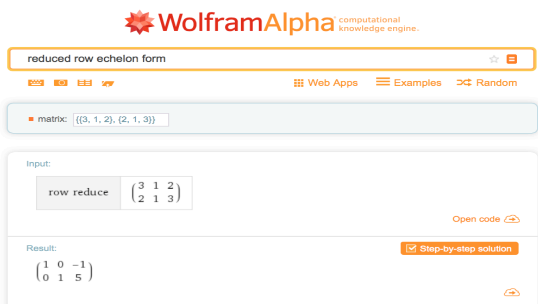
Math 2270-002
Homework due September 12.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; only the underlined problems need to be handed in. The Wednesday quiz will be drawn from all of these concepts and from these or related problems.

Note: If you are confident in your ability to compute reduced row echelon form of matrices you may use technology to do those computations - for any unaugmented matrix with more than 9 entries, and any augmented matrix with more than 12 entries. There are some problems in this assignment with fairly huge matrices where you should definitely use technology. Wolfram alpha, on your browser, will do these computations. Here's a screen shot that shows the syntax. If you do use technology, just copy the resulting reduced row echelon form and say which technology you used.



To make it easier for our grader, please order your homework so that all of your text problem solutions precede the ones I've created. I've listed the homework problems with this in mind.

Text:

1.7 Linear independence/dependence and the connection between this concept and homogeneous matrix equations $A\,\underline{\mathbf{c}} = \underline{\mathbf{0}}$.
1, **5**, **7**, **15**, **17**, **21**, **27**, **28**, **31**, 35, 36, 39, **42**.

1.8 Matrices represent special functions between Euclidean spaces, called "linear transformations."
1, 3, **5**, 7, 8, **9**, **11**, 13-15, **17**, 19, **21**, **27**, **37**.

1.9 All linear transformations $T : \mathbb{R}^n \to \mathbb{R}^m$ *are matrix transformations.*
1, **3**, **5**, **7**, **9**, **13**, **19**, **23**, **27**, **35**, **38**

Custom problems:

<u>w3.1</u> (section 1.7 Reduced row echelon form is unique because it encodes column dependencies of the original matrix.) Suppose $A = [\underline{\boldsymbol{a}}_1\ \underline{\boldsymbol{a}}_2\ \underline{\boldsymbol{a}}_3\ \underline{\boldsymbol{a}}_4\ \underline{\boldsymbol{a}}_5]$ is a matrix with 4 rows and 5 columns. Suppose we know that $\underline{\boldsymbol{a}}_1 \neq \underline{\mathbf{0}}$; $\{\underline{\boldsymbol{a}}_1, \underline{\boldsymbol{a}}_2\}$ is linearly independent; $\underline{\boldsymbol{a}}_3 = 3\,\underline{\boldsymbol{a}}_1 - 2\,\underline{\boldsymbol{a}}_2$; $\underline{\boldsymbol{a}}_4$ is not in the span of $\{\underline{\boldsymbol{a}}_1, \underline{\boldsymbol{a}}_2\}$; and $\underline{\boldsymbol{a}}_5 = 2\,\underline{\boldsymbol{a}}_1 - \underline{\boldsymbol{a}}_2 + 1.5\,\underline{\boldsymbol{a}}_4$.

<u>a</u> What matrix must the reduced row echelon form of A be?

<u>b</u> Can you determine whether or not the original five columns of A span $\mathbb{R}^4$? Explain.

<u>w3.2</u> (1.7-1.8)

<u>a</u> Suppose $\{\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \underline{\boldsymbol{v}}_3\}$ is a linearly dependent set in $\mathbb{R}^n$. Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation. Explain why $\{T(\underline{\boldsymbol{v}}_1), T(\underline{\boldsymbol{v}}_2), T(\underline{\boldsymbol{v}}_3)\}$ must be linearly dependent in $\mathbb{R}^m$.

<u>b</u> Suppose $\{\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \underline{\boldsymbol{v}}_3\}$ is a linearly independent set in $\mathbb{R}^n$. Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation. Do $\{T(\underline{\boldsymbol{v}}_1), T(\underline{\boldsymbol{v}}_2), T(\underline{\boldsymbol{v}}_3)\}$ have to be linearly independent in $\mathbb{R}^m$? Explain why, or give a counterexample and explain why not.

<u>w3.3</u> Classify each of the following as true or false and justify your choice.

<u>a</u> Assume the equation $A\,\underline{\mathbf{x}} = \underline{\boldsymbol{b}}$ has a solution. The solution is unique precisely when the equation $A\,\underline{\mathbf{x}} = \underline{\mathbf{0}}$ has only the trivial solution.

<u>b</u> Two non-zero vectors are linearly dependent if and only if they have the same span.

<u>c</u> If a set of vectors contains fewer vectors than there are entries in each vector, then the set is linearly independent.

<u>d</u> Every matrix transformation is a linear transformation.

<u>e</u> Suppose $T : \mathbb{R}^5 \rightarrow \mathbb{R}^2$ is a linear transformation and $T(\underline{\mathbf{x}}) = A\,\underline{\mathbf{x}}$ for some matrix A and every $\underline{\mathbf{x}} \in \mathbb{R}^5$. Then A has 5 rows and 2 columns.

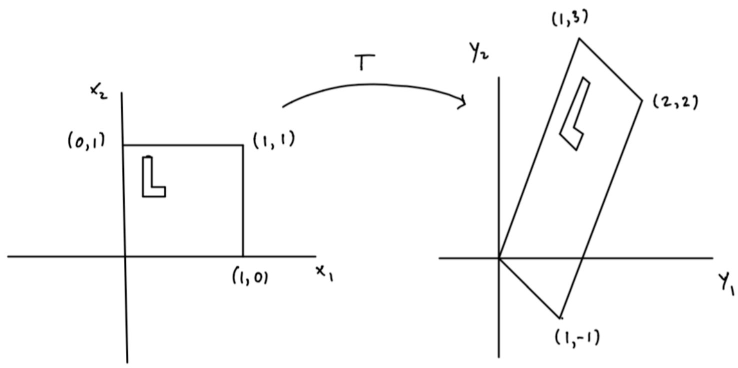
<u>w3.4</u> Give examples of the following:

<u>a</u> A 2×2 matrix A so that the solution set of the equation $A\,\underline{\mathbf{x}} = \underline{\mathbf{0}}$ is a line in $\mathbb{R}^2$ containing the point $(4, 1)$.

<u>b</u> A matrix B so that $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a solution of $B\,\underline{\mathbf{x}} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$.

w3.5

a Use the transformation picture for a mystery linear transformation $T(\underline{x}) = A\,\underline{x}$ to reconstruct the matrix A.



b) Sketch the line segment in the domain (above) whose position vectors are given parametrically by

$$L := \left\{ \begin{bmatrix} 0 \\ .5 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ 0 \le t \le 1 \right\}.$$

Then sketch the image line segment, $T(L)$, in the codomain (above) and provide a parametric formula for the image position vectors.