Math 2270-002
Homework 2, due Wednesday September 5 at the start of class.

Problems that are not underlined may be helpful for seeing if you can work with the underlying concepts. The underlined problems are the only ones to be handed in. Note: odd-numbered problems have the answers in the back of the book.

1.4 The matrix equation $A\,\underline{\boldsymbol{x}} = \underline{\boldsymbol{b}}$ encodes systems of linear equations as well as vector linear combination equations.
5, 7, **<u>11</u>**, **<u>13</u>**, **<u>17</u>**, **<u>19</u>**, **<u>21</u>**, 22, **<u>23</u>**, 24, 25, **<u>26</u>**, **<u>31</u>**.

<u>w2.1</u> Give examples of the following: (There are many such questions one can create. The purpose in having you answer them is to test and encourage your fluency in using and tying together the key ideas and definitions that we are learning.)

<u>a</u> A vector $\underline{\boldsymbol{v}}$ with no zero entries that is a linear combination of $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$.

<u>b</u> A 2 × 4 matrix whose columns do not span $\mathbb{R}^2$.
<u>c</u> A vector $\underline{\boldsymbol{b}}$ in $\mathbb{R}^3$ so that the matrix equation

$$\begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underline{\boldsymbol{b}}$$

has no solution.

1.5 Solutions sets of linear systems; homogeneous and non-homogeneous systems and how the solution sets correspond.
<u>5</u>, **<u>11</u>**,13, 17, 21, **<u>23</u>**, 27, 31.

<u>w2.2</u> Classify the following statements as true or false and justify your choice. (There are many such questions one can create. The purpose in having you answer them is to test and encourage your fluency in using and tying together the key ideas and definitions that we are learning.)

<u>a</u> If $\underline{\boldsymbol{x}}$ is a non-trivial solution of $A\,\underline{\boldsymbol{x}} = \underline{\boldsymbol{0}}$, then every entry of $\underline{\boldsymbol{x}}$ is non-zero.
<u>b</u> The equation $A\,\underline{\boldsymbol{x}} = \underline{\boldsymbol{b}}$ is homogeneous if the zero vector is a solution.
<u>c</u> The set $span\{\underline{\boldsymbol{u}}, \underline{\boldsymbol{v}}\}$ is always visualized as a plane through the origin.
<u>d</u> When $\underline{\boldsymbol{u}}$ and $\underline{\boldsymbol{v}}$ are non-zero, $span\{\underline{\boldsymbol{u}}, \underline{\boldsymbol{v}}\}$ contains the line through $\underline{\boldsymbol{u}}$ and the origin.

<u>w2.3</u> Suppose A is a 3 × 3 matrix and $\underline{\boldsymbol{b}}$ is a vector in $\mathbb{R}^3$ so that the equation $A\,\underline{\boldsymbol{x}} = \underline{\boldsymbol{b}}$ has a unique solution. Do the columns of A span $\mathbb{R}^3$? Explain.

<u>w2.4</u> In this problem you will consider the possible "shapes" of matrices in reduced row echelon form. For example, for 2 × 2 matrices in rref, there are either zero, one or two pivots. And depending on where the pivots are located, there are four resulting possible shapes:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & * \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Note that in the second matrix above we have used a $*$ for the second entry in the first row, to represent the fact that the entry there - in the non-pivot column - could be any real number.
<u>a</u>) Carry out a corresponding analysis for the possible reduced row echelon forms of 3×3 matrices. Hint: Making use of the 4 conditions that must be satisfied for matrices to be in reduced row echelon form, you will find that there are 8 possible shapes, which can organized by the number of pivots and in which columns they occur.
<u>b</u>) Without writing out all the possibilities, calculate (count) how many possible shapes there are for the reduced row echelon forms of 4×4 matrices. Hint: Consider whether each column is a pivot column or not.

1.6: *Some applications of the matrix equation* $A\,\underline{\mathbf{x}} = \underline{\mathbf{b}}$
<u>3</u>, <u>7</u>, <u>**11**</u> (in 11, the arrow with the number "80" should be pointing towards "C", not away from it.).

<u>**w2.5**</u> Find the equation of the parabolic graph that passes through the points $(x, y) = (1, 0), (2, 3), (4, 3)$: Exhibit the three linear equations in the three unknowns a, b, c, so that the parabola

$$y = a\,x^2 + b\,x + c$$

passes through the three indicated points, and then use Gaussian elimination to solve the system.

<u>**w2.6**</u> In class (and in the text) we discuss the balancing equation for complete combustion of propane. But if there is not enough oxygen present - e.g. if one burns propane (or any other hydrocarbon) without adequate ventilation - not all of the intermediate product carbon monoxide CO is converted into carbon dioxide. This yields a slightly more complicated equation to balance, namely

$$x_1\, C_3H_8 + x_2\, O_2 \;\rightarrow\; x_3\, CO_2 + x_4\, H_2O + x_5 CO.$$

Balance the equation. Set $x_1 = 1$ and let x_5 be the free parameter in the solutions. What interval of x_5 values make sense in this problem, i.e. so that all of the variables $x_1, x_2, \ldots x_5$ are non-negative? Explain.