Math 2270-002
Week 13-14 homework,
due November 28.

6.5 Least square solutions
1, **3**, 5, **7**, **9**, 11, 15, **17**, 19

6.6 Linear models for data fitting
1, **7**, and exercise **w13.2** below about the human height-weight power law.

6.7 Inner product spaces
6.7.25 extended (Legendre polynomials): For functions in $C[-1, 1]$ Use Gram-Schmidt to find an orthogonal basis for $W = span\{1, t, t^2, t^3\}$, with respect to the inner product

$$\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)\, dt.$$

In the first part of the problem scale the orthogonal polynomials so that the coefficient of the leading power of t is 1. Then normalize the orthogonal basis to make it orthonormal. You can read more about Legendre polynomials at Wikipedia.

w13.1 In quiz 13 you found $proj_W\, \underline{b}$, for $\underline{b} = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$ and $W = span\left\{\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -4 \end{bmatrix}\right\}$, by first finding an orthogonal basis for W and then using that basis to do the projection. Rework this projection problem by using the method of least squares algorithm from section 6.5, as we've also discussed in class.

Math 2270-002 Fall 2018

A Power Law For Human Heights and Weights

Body Mass Index

A person's BMI is computed by dividing their weight by the square of their height, and then multiplying by a universal constant. If you measure weight in kilograms, and height in meters, this constant is the number one. If you measure height in inches and weight in pounds then the formula is

$$BMI = 703\frac{w}{h^2}$$

The graph of heights and weights for which BMI has a constant value B is the parabola

$$w = \frac{B}{703}\cdot h^2.$$

Thus, the assumption underlying BMI is that for adults at equal risk levels (but different heights), weight should be proportional to the square of height. This is a historical accident and at some point became a dogma. The BMI was popularized in the 1960's in the U.S., by proponents who were initially unware that they were repeating history. It is easy to deduce that if people were to scale equally in all directions when they grew, weight would scale as the cube of height. That particular power law seems a little high, since adults don't look like uniformly expanded versions of babies; we seem to get relatively stretched out length-wise when we grow taller. One would expect the best predictive power to be somewhere between 2 and 3. If the power is much larger than 2 then one could argue that the body mass index might need to be modified to reflect this fact.

It turns out a Belgian demographer, Adolphe Quetelet, also called the "Father of Statistics", originally proposed a power of p=2 for adults, based on his own data analysis during the early 1800's. In a footnote which history has forgotten, he said that a power of 2.5 is more appropriate if you want an approximation for people of all ages. He actually wrote that the square of the weight should scale like the fifth power of the height, because pre-calculators, fractional powers were harder for people to deal with. My recollection is that this footnote appears in the 1835 publication "Sur l'homme et le développement de ses facultés, ou Essai de physique sociale". I have read the footnote.

There is (or at least there was, 20 years ago) a database at the U.S. Center for Disease Control, of national body data collected between 1976 and 1980. From this data I have extracted the median heights and weights for boys and girls, age 2-19. The national data is shown on the next page; heights are given in inches and weights are in pounds.

w13.2) Find the power law

$$w = C\,h^p$$

predicted by this data, by finding a least squares line fit to the ln-ln data. (Combine the boy-girl data into one set.) We will discuss this further in class on Monday after Thanksgiving. Note that if such a power law holds, taking logarithms of both sides of the identity yields

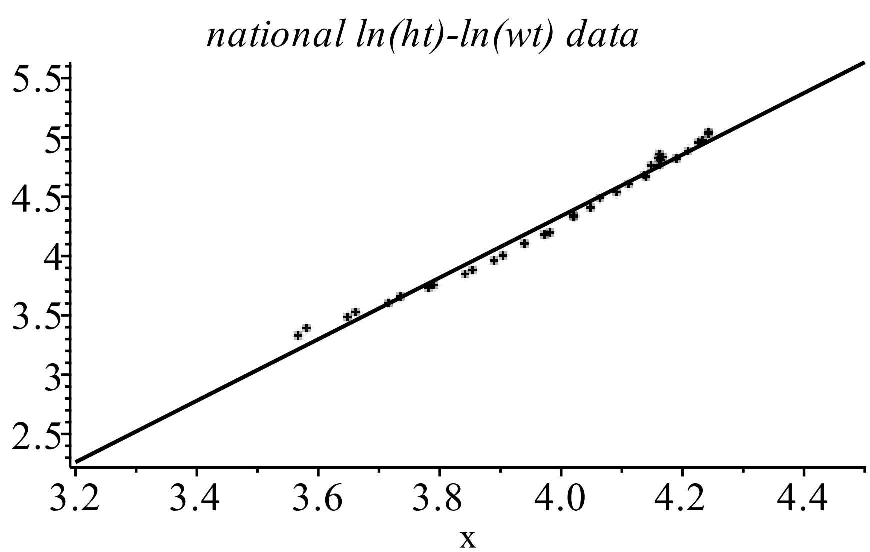
$$\ln(w) = \ln(C) + p\cdot\ln(h).$$

If we write $Y = \ln(w)$, $X = \ln(h)$ then this is the equation of a line in the $X - Y$ plane, where the slope is the original power p and the Y-intercept equals $\ln(C)$,

$$Y = Y_0 + p\,X$$

age	boy height	weight	girl height	weight
2	35.9	29.8	35.4	28.0
3	38.9	34.1	38.4	32.6
4	41.9	38.8	41.1	36.8
5	44.3	42.8	43.9	41.8
6	47.2	48.6	46.6	47.0
7	49.6	54.8	48.9	52.5
8	51.4	60.8	51.4	60.8
9	53.6	66.5	53.1	65.5
10	55.7	76.8	55.7	76.1
11	57.3	82.3	58.2	89.0
12	59.8	93.8	61.0	100.1
13	62.8	106.8	62.6	108.1
14	66.0	124.3	63.3	117.1
15	67.3	132.6	64.2	117.6
16	68.4	142.1	64.3	122.6
17	68.9	145.1	64.2	128.8
18	69.6	155.3	64.1	124.5
19	69.6	153.2	64.5	126.0

A graph of the best line fit to the national $\ln - \ln$ data. It's a pretty good fit! (Infants are a little heavier than the line predicts, adolescent data is slightly below the line, and as adults mature they rise a bit above the line. The slope of the line will be the power in the approximate power law.



<u>submission</u>: I prefer that you use Matlab. In that case, submit a script to CANVAS which computes the least squares line fit; which recovers the power law; and which creates a graph of the log-log point scatterplot together with the least squares line (as above); and a separate plot which combines a scatter plot of the original height-weight data, together with the graph of the power law function. We will use an analogous script for a smaller problem in class on Monday. If you don't use Matlab please hand in hard copies of same results with the rest of your homework.