Math 2270-002

Homework due November 14.

Appendix B and class notes on complex numbers

6.1 dot product, length, orthogonality

1, 3, **5**, **7**, **9**, **11**, **13**, 15, **17**, **19**, **23**. (In 23, do note that the Pythagorean Theorem holds.)

6.2 orthogonal projection onto lines (through the origin).

11, 13, 15

w11.1 (geometric meaning of multiplication and multiplicative inverses in the complex plane) In class Friday we talked about how when you multiply two complex numbers their polar angles add and their moduli multiply. Using Euler's formula, the complex number written in polar form can be expressed using exponentials,

$$r\,(\cos\theta + i\sin\theta) = r\,e^{i\theta}$$

and the multiplication property can be written as

$$r\,e^{i\theta}\,\rho\,e^{i\phi} = r\,\rho\,e^{i(\theta+\phi)}.$$

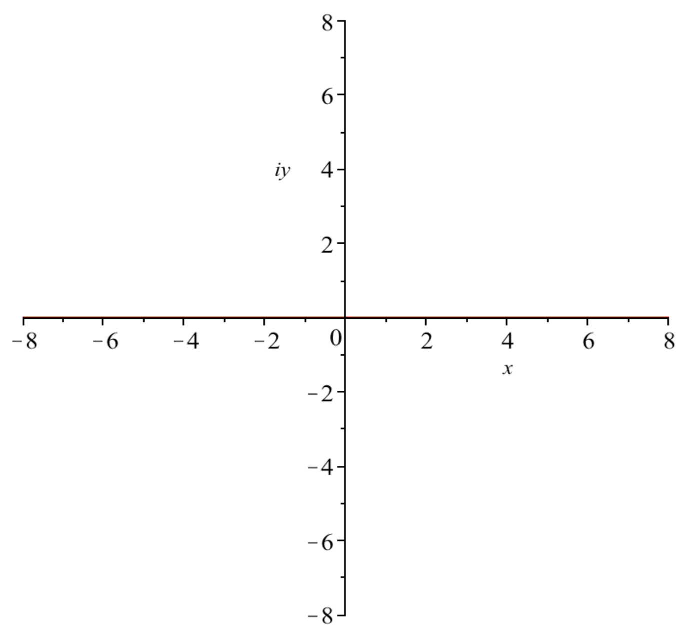
a Express $z = \sqrt{3} + i$ in polar form, $z = r\,[\cos\theta + i\sin\theta] = r\,e^{i\theta}$ (Hint: its polar angle is an elementary angle).

b Compute z^3 two ways, first using $z = \sqrt{3} + i$, and then using $z = r\,e^{i\theta}$. (You will use $e^{\frac{i\pi}{2}} = i$.)

c Find $\frac{1}{z}$ using the conjugate formula, $\frac{1}{z} = \frac{\overline{z}}{|z|^2}$.

d On the other hand, $\frac{1}{z} = \frac{1}{r}e^{-i\theta}$ since $r\,e^{i\theta}\,\frac{1}{r}\,e^{-i\theta} = 1$. Verify that the polar form of the inverse agrees with your work in c.

e Illustrate your work in a,b,c,d on the complex plane.



w11.2 By using the algebra of dot products verify the *parallelegram identity* in $\mathbb{R}^n$ which asserts that the sum of the squared lengths of the diagonals of a parallelgram equal the sums of the squared lengths of the four sides:

$$\|\underline{\boldsymbol{u}} + \underline{\boldsymbol{v}}\|^2 + \|\underline{\boldsymbol{u}} - \underline{\boldsymbol{v}}\|^2 = 2\|\underline{\boldsymbol{u}}\|^2 + 2\|\underline{\boldsymbol{v}}\|^2 .$$

w11.3 Consider the line L in $\mathbb{R}^2$ with position vectors given as $L = span\left\{\begin{bmatrix} -3 \\ 1 \end{bmatrix}\right\}$, and the point with position vector $\underline{P} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.

a Find the orthogonal projection of $\underline{\boldsymbol{P}}$ onto L, i.e. $proj_L\underline{\boldsymbol{P}}$.

b Express $\underline{\boldsymbol{P}}$ as the sum a vector in L with a vector $\underline{z}$ perpendicular to L.

c What is the (nearest point) distance from (the point with position vector) $\underline{\boldsymbol{P}}$ to L ?

d Pick any point on L which is neither the origin nor $proj_L\underline{\boldsymbol{P}}$. Verify that the Pythagorean Theorem holds for the triangle determined by the points (with position vectors) $\underline{\boldsymbol{P}}$, $proj_L\underline{\boldsymbol{P}}$, and the point you chose.

e Make a diagram which illustrates all of your work from a,b,c,d.