Math 2270-002
Homework due November 7.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; only the underlined problems need to be handed in. The Wednesday quiz will be drawn from all of these concepts and from these or related problems. (But we won't have a Wednesday quiz this week, since our midterm is on Friday.)

5.3: Matrix diagonalization
1, 3, 9, 11, 13, **21**, **23**, **24**, **25**, 29, 31.

5.4 Matrices for linear transformations as a framework to understand change of basis, diagonalization, similar matrices, and more.
1, **3**, **5**, **11**, **13**, **17**.

5.5 Complex eigenvalues and eigenvectors. 2 by 2 matrices with complex eigendata are similar to rotation-dilation matrices.
1, **7**, **11**, **13**.

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Here are the matrices you worked with in last week's custom homework problems, when you were asked to find eigenvalues and eigenspace bases. I have included the answers, which you will use in this week's first custom problem. I show the factored the characteristic polynomials and eigenspace bases for each matrix. (For detailed explanations if you were confused, consult the homework 9 solutions on CANVAS.)

w9.1a) $A := \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix}$ $p(\lambda) = (\lambda - 3)(\lambda - 1)$, $E_{\lambda=3} = span\left\{\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right\}$, $E_{\lambda=1} = span\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$.

w9.1b) $B := \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$ $p(\lambda) = (\lambda - 2)^2$, $E_{\lambda=2} = span\left\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$.

w9.1c) $C := \begin{bmatrix} 2 & 9 & 3 \\ -2 & -5 & 0 \\ 2 & 6 & 1 \end{bmatrix}$ $p(\lambda) = = -(\lambda - 1)(\lambda + 1)(\lambda + 2)$, $E_{\lambda=1} = span\left\{\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}\right\}$, $E_{\lambda=-1} = span\left\{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}\right\}$,

$E_{\lambda=-2} = span\left\{\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}\right\}$.

w9.1d) $E := \begin{bmatrix} 1 & 6 & 6 \\ 0 & -1 & -2 \\ 0 & 4 & 5 \end{bmatrix}$ $p(\lambda) = -(\lambda - 1)^2(\lambda - 3)$, $E_{\lambda=1} = span\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}\right\}$, $E_{\lambda=3} = span\left\{\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}\right\}$.

w9.1e) $F := \begin{bmatrix} 5 & 3 & -9 \\ -4 & -5 & 4 \\ 4 & 2 & -7 \end{bmatrix}$ $p(\lambda) = -(\lambda + 1)\cdot(\lambda + 3)^2$, $E_{\lambda=-1} = span\left\{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}\right\}$, $E_{\lambda=-3} = span\left\{\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}\right\}$.

w10.1) (These questions relate to section 5.3.)

a) Which of the matrices in 9.1 are diagonalizable and which are not?

b) For which of the matrices above could you have concluded diagonalizability just from the factored form of the characteristic polynomial, i.e. before actually finding the corresponding eigenspace bases? Hint: See Theorem 6 on page 286.

c) For the diagonalizable matrices *A*, *C* above, verify the diagonalizing identity $P^{-1}A\,P = \mathrm{D}$ by checking the equivalent and easier to check equation $A\,P = P\,\mathrm{D}$.

d) Explain what happens to the diagonal matrix D of eigenvalues for a diagonalizable matrix, if you change the order of the eigenvector columns in *P*, in the identity $A\,P = P\,\mathrm{D}$. Explain your conclusion.

w10.2 Compute A^{10} for the matrix *A* in w9.1a, without technology. (Of course, you may check your answer with technology.)

w10.3 Constructions. Give examples of the following, or explain why the requested construction is impossible. In this problem all matrices are assumed to be square, i.e. $n \times n$. I recommend finding the simplest possible examples you can think of.

a) A matrix which is invertible but not diagonalizable.

b) A matrix which is diagonalizable but not invertible.

c) A matrix which is invertible but for which $\lambda = 0$ is an eigenvalue.

d) An $n \times n$ matrix with less than *n* eigenvalues which is diagonalizable.

e) An $n \times n$ matrix with *n* distinct eigenvalues which is not diagonalizable.

w10.4 (This question relates to 5.4) Consider the matrix *A* of w9.1a),

$$A = \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix},$$

and the matrix transformation $T(x) = A\,x$ from $\mathbb{R}^2$ to $\mathbb{R}^2$. Find the matrix for *T* with respect to the eigenvector basis $\beta = \left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$, i.e. find $[T]_\beta$.

<u>Extra credit problem for 10 points</u>: Find the google-ranking of the 12 football teams in the Pac 12, based on the inter-league results through the weekend of November 3-4. Use Matlab. I will post a matlab script which produced the following ranking as of last weekend. So your major task is to figure out how to open and begin to use Matlab. It's available for free on all University computers I'm aware of which allow student accounts. I will provide further help as needed, or please also ask among your classmates and friends - and youtube can get you pretty far when you get stuck.

Here is the stochastic matrix as of November 1 - without google fudge factor. (screenshot from Matlab script):

```
%     AZ   AZS CAL CU   OR   OSU STA UCLA USC UT WA   WAS
SMa=[ 0,   0, 1/3.,0, 1/3,.25, 0,   0,  0,   0, 0,   0;     %AZ
      0,   0,  0,  0,  0, .25, 0,   0  1/3, 0, 0,   0;     %AZS
      0,   0,  0,  0,  0, .25, 0,   0,  0,   0,.5,   0;     %CAL
      0, 1/3 ,0,  0,  0,  0,   0,  1/3, 0,   0, 0,   0;     %CU
      0,   0, 1/3  0,  0,  0,   0,   0,  0,   0,.5,   0;     %OR
      0,   0,  0, 1/3, 0,  0,   0,   0,  0,   0, 0,   0;     %OSU
      0, 1/3, 0,  0, 1/3, 0,   0,   0, 1/3, 0, 0,   0;     %STA
    1/3, 0, 1/3, 0,  0,  0,   0,   0,  0,   0, 0,   0;     %UCLA
    1/3, 0,  0, 1/3, 0,  0,   0,   0,  0,   0, 0,   1;     %USC
    1/3, 0,  0,  0,  0,  0, .5,  1/3,1/3, 0, 0,   0;     %UT
    0,   1/3, 0, 1/3, 0,  0,   0,  1/3, 0, .5, 0,   0;     %WA
    0,   0,  0,  0, 1/3,.25,.5,   0,  0, .5, 0,   0;]    %WAS
```

After adding the google fudge factor to make the matrix regular stochastic, and then raising to a large power one sees that the vector of relative ranks places USC #1 in the league, ahead of Washington State #2 and Utah #3. The national rankings currently place Washington State at #8 and Utah at #15, with USC unranked. The national rankings are based on complete records of all teams and may come to their conclusions based on different algorithms. USC does well in our experiment because they are the only team so far to beat the otherwise top-ranked Washington State. Here is the google matrix, raised to the 20th power:

ans | SMa | SMb | SM

12x12 double

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.0579	0.0579	0.0579	0.0579	0.0579	0.0579	0.0579	0.0579	0.0579	0.0579	0.0579	0.0579
2	0.0619	0.0619	0.0618	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0618
3	0.0644	0.0644	0.0644	0.0644	0.0644	0.0644	0.0644	0.0644	0.0644	0.0644	0.0644	0.0644
4	0.0434	0.0434	0.0434	0.0434	0.0434	0.0434	0.0434	0.0434	0.0434	0.0434	0.0434	0.0434
5	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774
6	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248
7	0.0960	0.0960	0.0960	0.0960	0.0960	0.0960	0.0960	0.0960	0.0960	0.0960	0.0960	0.0960
8	0.0472	0.0472	0.0472	0.0472	0.0472	0.0472	0.0472	0.0472	0.0472	0.0472	0.0472	0.0472
9	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556
10	0.1272	0.1272	0.1272	0.1272	0.1272	0.1272	0.1272	0.1272	0.1272	0.1272	0.1272	0.1272
11	0.1097	0.1097	0.1097	0.1097	0.1097	0.1097	0.1097	0.1097	0.1097	0.1097	0.1097	0.1097
12	0.1346	0.1346	0.1346	0.1346	0.1346	0.1346	0.1346	0.1346	0.1346	0.1346	0.1346	0.1346
13												
14												