Math 2270-002

Homework 1, due August 29 at the start of class.

Problems that are not underlined may be helpful for seeing if you can work with the underlying concepts. The underlined problems are the only ones to be handed in. Note: odd-numbered problems have the answers in the back of the book.

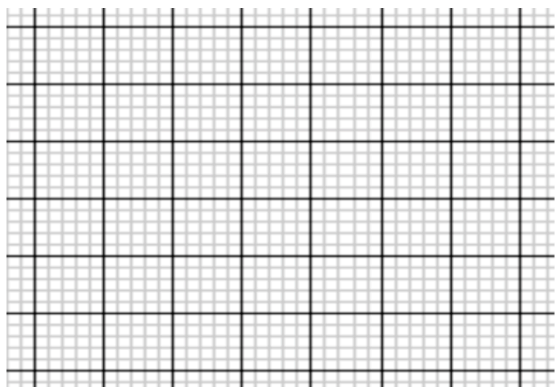
1.1 Solve linear systems using augmented matrices and Gaussian elimination with elementary row operations. Interpret results geometrically and understand consistent vs. inconsistent systems.

text problems: 1, 3, **11**, **13**, 15, **17**, 19, **23**, **29**, **31**, **33**.

w1.1 Consider the system of two equations in two unknowns.

$$\begin{aligned} x + 3y &= 5 \\ -3x + 2y &= -4. \end{aligned}$$

a Sketch the solution set lines to each equation, and estimate the point of intersection geometrically.



b Solve the system of equations and verify that your algebraic solution corresponds to the point you found in w1.1a. You may use Gaussian elimination or the method of substitution.

w1.2a Recall that we call a linear system of equations *inconsistent* if it has no solution(s). Give an example of a linear system of two equations in two variables that is inconsistent. (Hint: Thinking geometrically may help.)

b Give an example of a linear system of two equations in two variables that has infinitely many solutions.

c Give an example of a linear system of three equations in two variables that has exactly one solution.

d Is it possible to have a linear system of two equations in three variables that has exactly one solution? Hint: you may reason geometrically or algebraically. If you reason algebraically it will be helpful to think about the reduced row echelon form of the reduced matrix.

1.2 Use Gaussian elimination along with row echelon and reduced row echelon forms of augmented matrices to understand solutions sets to systems of linear equations.

text problems: 3, 7, **11**, **13**, 15, 19, **21**, 23, **25**, **27**, **31** .

w1.3 Consider the following two systems of equations

$$\begin{aligned} 2x + 2y &= 2 \\ -3x + 3z &= -6 \\ x + 4y + 3z &= -2 \end{aligned} \qquad\qquad \begin{aligned} 2x + 2y &= 0 \\ -3x + 3z &= 0 \\ x + 4y + 3z &= 1 \end{aligned}$$

Note that the left sides of these two systems are the same - only the right hand sides are different.

a Exhibit a single augmented matrix that lets you solve both systems at once. Hint: it has 5 columns.
b Compute the reduced row echelon form of the augmented matrix in part (a) and then write down all of the solutions for each system. Hint: One system has infinitely many solutions and the other system has none.
c Is there any right hand side vector ***b*** for which the system

$$\begin{aligned} 2x + 2y &= b_1 \\ -3x + 3z &= b_2 \\ x + 4y + 3z &= b_3 \end{aligned}$$

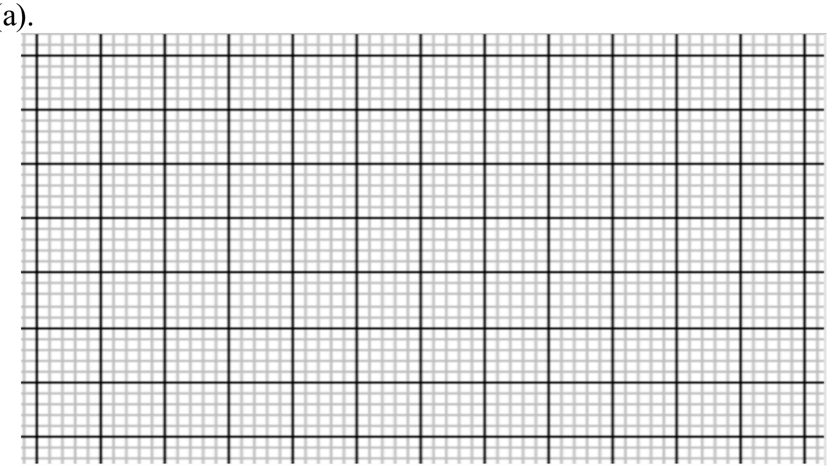
has exactly one solution? Explain why or why not.

1.3: *Solve vector equations by interpreting them as systems of linear equations.*
text problems: 1, 3, 5, **7**, **9**, 11, **13**, **15**, **19**, **27**, **29**.

w1.4 (This is like exercises 4,5 in Friday August 24 class notes.) Consider the three vectors in $\mathbb{R}^2$:

$$\boldsymbol{u} := \begin{bmatrix} 3 \\ -2 \end{bmatrix} \quad \boldsymbol{v} := \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \boldsymbol{w} := \begin{bmatrix} 9 \\ 2 \end{bmatrix}$$

a Express ***w*** as a linear combination of ***u*** and ***v***, by solving the appropriate vector equation.
b) Make a careful and accurate sketch which illustrates your answer to (a). You may use your own graph paper or cut out the piece below and tape it into your homework.
c) Find a linear combination of ***u***, ***v***, ***w*** which adds up to the zero vector. (You've already done the work for this in part (b), if you just rearrange your equation!) Illustrate this linear combination adding up to zero on your sketch for (a).



w1.5 (This is like Exercise 5 in Friday August 24 notes.) Let

$$\boldsymbol{u} := \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \boldsymbol{v} := \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}, \quad \boldsymbol{w} := \begin{bmatrix} -5 \\ 5 \\ 5 \end{bmatrix}.$$

a) Use a reduced row echelon computation to check that the span of these three vectors is not $\mathbb{R}^3$.
b) Use your reduced row echelon form computation to write ***w*** as a linear combination of ***u***, ***v***. (Hint: what augmented matrix would you have if you were solving $c_1\boldsymbol{u} + c_2\boldsymbol{v} = \boldsymbol{w}$ for c_1, c_2 ?)
c) The span of these three vectors is actually a plane through the origin. Find an implicit equation $a\,x + b\,y + c\,z = 0$ satisfied all points (x, y, z) whose position vectors $[x, y, z]^T$ are in the plane. Hint: You are looking for the condition on the vector ***b*** so that it is a linear combination of ***u***, ***v***. In other words,

for which $\underline{\boldsymbol{b}}$ is the system

$$c_1\underline{\boldsymbol{u}} + c_2\underline{\boldsymbol{v}} = \underline{\boldsymbol{b}}$$

consistent?