Math 2270-3
Tuesday Sept. 8

- do ~~reflections~~ projections & reflections from §2.2 (Wed & Friday notes) (we did rotations.)

- Discuss affine transformations, which are the composition of a matrix transformation with a translation (Friday notes pags 2-3.)

- §2.3 Matrix multiplication:

Suppose $T_1 : \mathbb{R}^n \to \mathbb{R}^m \quad T_1(\vec{x}) = A\vec{x} \qquad (A_{m\times n})$
and $T_2 : \mathbb{R}^m \to \mathbb{R}^p \quad T_2(\vec{y}) = B\vec{y} \qquad (B_{p\times m})$

are matrix transformations.

- Then the composition $T_2 \circ T_1 : \mathbb{R}^n \to \mathbb{R}^p$ is linear:

$$\text{(a)}\quad T_2(T_1(\vec{u}+\vec{v})) = T_2(T_1(\vec{u}) + T_1(\vec{v})) \qquad T_1 \text{ linear}$$
$$= T_2\circ T_1(\vec{u}) + T_2\circ T_1(v) \qquad T_2 \text{ linear}$$

$$\text{(b)}\quad T_2(T_1(k\vec{v})) = T_2(kT_1(\vec{v})) \qquad T_1 \text{ linear}$$
$$= k\, T_2(T_1(\vec{v})) \qquad T_2 \text{ linear}$$

- So $T_2 \circ T_1$ is also a matrix transformation.
- Its j^{th} column is
$$T_2(T_1(\vec{e}_j))$$
$$= T_2(A\vec{e}_j)$$
$$= T_2(col_j(A))$$
$$= B(col_j(A))$$

i.e. the matrix of $T_2 \circ T_1$ is

$$\left[\, B\,col_1(A) \,\middle|\, B\,col_2(A) \,\middle|\, \ldots\, B\,col_n(A) \,\right] := BA$$

i.e. For $B_{p\times m}$, $A_{m\times n}$ then $BA_{p\times n}$ is defined by

$$entry_{ij}(BA) = row_i(A)\cdot col_j(B)$$

or equivalently,

$$col_j(BA) = B\,col_j(A)$$

and $(BA)\vec{x} = B(A\vec{x})$, because BA is the matrix of $T_2\circ T_1$!

Example 1 Compute
$$\begin{array}{cc} B & A \\ \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} & \begin{bmatrix} 0 & 1 & -2 & 3 \\ 1 & 0 & 4 & 1 \end{bmatrix} \end{array}$$

Example 2 for $T_1\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 & 3 \\ 1 & 0 & 4 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

$$T_2\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Compute $(T_2 \circ T_1)\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right)$. How does this relate to Example 1?

Example 3 Recall from last Friday that the linear transformation which rotates counterclockwise by angle α has matrix

$$[Rot_\alpha] = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

and $[Rot_\beta] = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$.

Compute $[Rot_\beta][Rot_\alpha] = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$. What do you deduce?!

Matrix algebra! (Matrix addition is defined like vector addition, entry ij $(A+B) = a_{ij} + b_{ij}$

Check satisfies commutative & associative properties, see below)

(1) $(AB)C = A(BC)$

hint: use page 1

(2) $AB \neq BA$ in general!

(3) $A(B+C) = AB + AC$

$(A+B)C = AC + BC$

oops, forgot matrix addition and scalar mult axioms, which are just the same as for vectors

(i) $A+B = B+A$

(ii) $(A+B)+C = A+(B+C)$

(iii) $k(A+B) = kA + kB \qquad k \in \mathbb{R}$

(iv) $(k_1+k_2)A = k_1A + k_2A \qquad k_1, k_2 \in \mathbb{R}.$

<u>tomorrow</u>: inverse matrices; maybe intro to fractals.