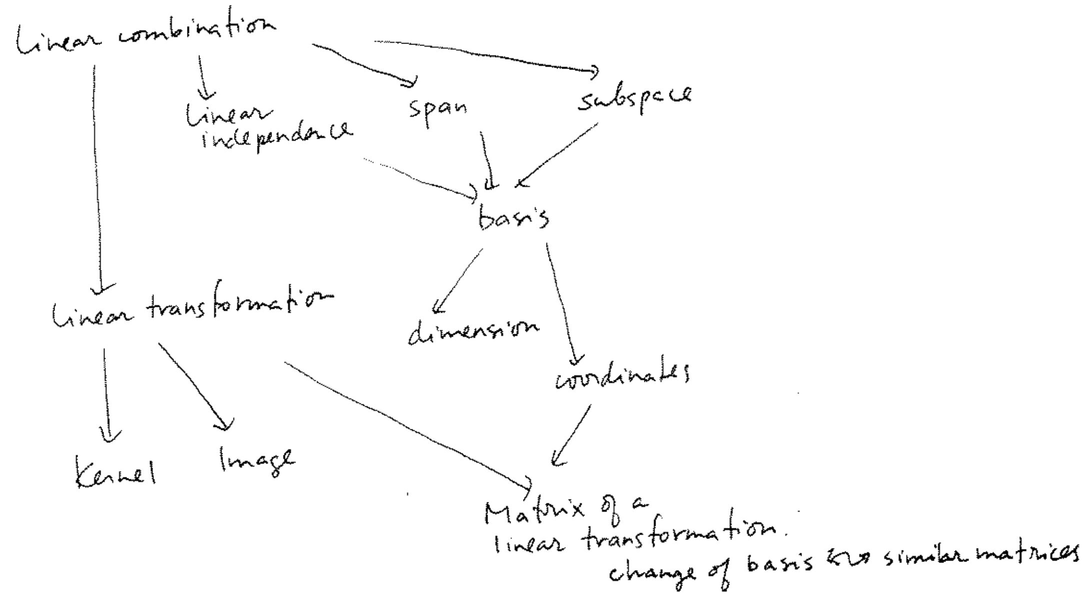
Math 2270-3
Wednesday 30 Sept.

Finish pages 3, 4 Tuesday: <u>similar matrices</u>
Then begin §4.1: <u>Linear Spaces</u> (also called <u>Vector spaces</u>, in most books)

<u>In chapter 3</u> we followed this tree of ideas for $\mathbb{R}^n$: (p. 156 text)



A <u>linear space</u> (or <u>vector space</u>) is a collection of objects for which you can take linear combinations, and so that the expected algebraic properties (below) hold. (p. 154 text).
It turns out there are many more of these spaces than $\mathbb{R}^n$! <u>And the same diagram holds!</u>

<u>Def</u>: A linearspace V is a set endowed with a rule for addition (If $f, g \in V$ then so is $f+g$) and a rule for scalar multiplication (If $f \in V$ and $k \in \mathbb{R}$ then $kf \in V$), such that these operations satisfy the following 8 rules
($\forall\ f, g, h \in V$, $\forall\ c, k \in \mathbb{R}$)

1. $(f+g)+h = f+(g+h)$
2. $f+g = g+f$
3. $\exists$ neutral element $n \in V$ s.t. $f+n = f\ \forall f \in V$. This n is unique and is denoted by 0
4. $\forall f \in V\ \exists\ g \in V$ s.t. $f+g=0$. This g is unique and is denoted by $-f$
5. $k(f+g) = kf + kg$
6. $(c+k)f = cf + kf$
7. $c(kf) = (ck)f$
8. $1f = f$.

Lots of examples!
See text p. 154–162.

Example 3 $\quad F(\mathbb{R},\mathbb{R}) = \{ f : \mathbb{R} \to \mathbb{R} \}$

$$(f+g)(x) := f(x) + g(x)$$
$$(kf)(x) := k \cdot f(x)$$

Verify properties!
What's the neutral element?

Example 4 $\quad \mathbb{R}^{m\times n} := \{ M_{m\times n} \text{ , i.e. all } m\times n \text{ matrices} \}$.
addition & scalar multiplication of matrices

Example 5 $\quad$ The set of all infinite sequences of real numbers

$$(x_0, x_1, x_2, x_3, \dots) + (y_0, y_1, y_2, \dots) := (x_0+y_0,\ x_1+y_1,\ x_2+y_2, \dots)$$
$$k\,(x_0, x_1, x_2, \dots) := (kx_0, kx_1, kx_2, \dots)$$

Example 8 $\mathbb{C} = \{a + ib \text{ s.t. } a, b \in \mathbb{R}\}$.

$(i = \sqrt{-1})$.
$i^2 = -1$
although we don't multiply complex numbers when we consider $\mathbb{C}$ as a real scalar vector space

$(a+ib) + (c+id) := (a+c) + i(b+d)$

$k(a+ib) := ka + ikb \qquad k \in \mathbb{R}$

For a linear space V, can you define

- linear combination of $f_1, \ldots f_n$:

- a subspace W (and notice W itself is a linear space!)

- span $\{f_1, \ldots f_n\}$

- $\{f_1, \ldots f_n\}$ linearly independent
  linearly dependent

- $\{f_1, \ldots f_n\}$ are a basis for V:
  coords for $v \in V$ w.r.t basis $\mathcal{B} = \{f_1, \ldots f_n\}$.

- $\dim V$

<u>Example</u> $\mathbb{P}_2 := \{ p(x) = a_0 + a_1 x + a_2 x^2 : a_i \in \mathbb{R} \} \subset \mathbb{F}(\mathbb{R}, \mathbb{R})$

(many more examples of subspaces of $\mathbb{F}(\mathbb{R},\mathbb{R})$ – see example 12)

Show $\mathbb{P}_2$ is a subspace of $\mathbb{F}(\mathbb{R}, \mathbb{R})$

Find a "natural" basis.

Is $\{1+x, 1+x^2, x+x^2\}$ another basis?

Find coords of $p(x) = x^2$ with respect to both bases