9/25 = (1)

Math 2270-3
Friday Sept 18
§3.2-3.3

I'll be in LCB 115 Saturday 11-1

HW for Friday Sept 25 (Exam 1!)
3.2 1 4 5 7 8, 14, 15, 21, 28, 33, 45
3.3 6 7 18, 21 25: use Maple or other!
29, 32, 36, 37
3.4 1, 3 5 6, 11, 14, 15 16, 17, 21, 22, 31
33, 43, 53, 56

- Warm-up (1)
- finish subspace discussion, pages 4-5 Tuesday
- review definitions (and complete) pages 1-2 Wednesday, as we do the big Maple example, pages 3-5 Wed, enhanced in today's notes (on-line version)
- discuss why reduced row echelon form is unique. (2)
- (on your own) useful Maple commands (3)

Warm-up:

a) Exhibit some linear transformations with image

(i) the line $L = \{t\begin{bmatrix}1\\3\end{bmatrix}, t \in \mathbb{R}\}$ in $\mathbb{R}^2$

(ii) the plane $y_1 + 2y_2 - y_3 = 0$ in $\mathbb{R}^3$

b) Exhibit some linear transformations with kernel

(i) the line $L = \{t\begin{bmatrix}1\\3\end{bmatrix}, t \in \mathbb{R}\}$ in $\mathbb{R}^2$

(i) the plane $x_1 + 2x_2 + x_3 = 0$ in $\mathbb{R}^3$.

Why reduced row echelon form of a matrix is unique!
(After all, there are so many different orders in which you could reduce a matrix, why do you always end up with the same rref(A), assuming you make no algebra errors?)

The reason is that the solution set to $A\vec{x} = \vec{0}$ stays the same as you do elementary row operations to the augmented matrix. Each solution $\vec{x}$ to $A\vec{x} = \vec{0}$ is a dependency relation

$$x_1 \, col_1(A) + x_2 \, col_2(A) + \dots + x_n \, col_n(A) = \vec{0}$$

on the columns of A, and vise verse. The dependency relations on A's columns are therefore exactly those on the columns of rref(A). But these dependency relations completely determine the entries of rref(A).

One can write down a general proof, but it might be more convincing to continue with the Maple example from Wed:

$$A := \begin{bmatrix} 1 & 2 & 3 & -1 & -2 & 6 \\ 0 & 1 & 1 & -2 & -1 & 3 \\ 2 & -4 & -2 & -1 & 4 & 3 \\ 3 & -2 & 1 & -2 & 2 & 9 \end{bmatrix}$$

> *ReducedRowEchelonForm*(*A*);

first, consider this to be a rref(A). The reasoning below explains why it's the only possible rref(A)!

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

>

$col_1(A) \neq \vec{0} \implies col_1(rref(A)) = \vec{e}_1$

$col_2(A)$ not dependent on $col_1(A)$ $\implies col_2(rref(A)) = \vec{e}_2$

$col_3(A) = col_1(A) + col_2(A) \implies col_3(rref(A)) = col_1(rref(A)) + col_2(rref(A)) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$col_4(A)$ not dependent on $col_1(A), col_2(A)$ (or $col_3(A)$) $\implies col_4(rref(A)) = \vec{e}_3$

$col_5(A) = -col_2(A) \implies col_5(rref(A)) = -col_2(rref(A)) = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

$col_6(A) = 3\,col_1(A) + col_2(A) - col_4(A) \implies col_6(rref(A)) = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 0 \end{bmatrix}$

Introduction to Matrices and Linear Algebra on Maple
Math 2270–3
September 2, 2009 – updated September 16 – posted on our lecture page

With Maple you can create combined text–math documents. Alternately, you can create math output and export it to other documents. This text field was created using the "T" menu item. To do math in a document, move your cursor to just above the place you want to do it, and use the "[>" menu item – a math field will be created as near to directly below your current cursor position as possible. If you're in a Math field and want text below it, hit the "T" menu item.

```
> 3 + 4; 5·7;
  6·5 : #commands end with ";" if you want to see output, with ":" if you don't
        #commands are executed as soon as you hit the "enter" or "return" key.
        # to enter multiple command lines, hold down the shift key while you hit return
>
```

The newer Maple package for matrix and linear algebra is called "LinearAlgebra". Commands in LinearAlgebra tend to be concatenated capitalized words for the complete name of the procedure you wish to do. There are also shortcuts.

Load the LinearAlgebra package:

```
> with(LinearAlgebra) : # to see commands, end with ";" to hide them, use ":"
```

Define a matrix:

```
> A := Matrix(3, 4, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]);
      # a 3 by 4 matrix, with entries moving left to right, and down the rows
      # ":=" is how we define things in Maple
> A;
> B := Matrix([[1, 2, 3], [1, 2, -1], [0, 2, -1]]);
      # another format – enter each row
>
```

Compute the reduced row echelon form:

```
> ReducedRowEchelonForm(A);
```

Define a (column) vector:

```
> b := Vector([2, 2, 2]);
```

Augment a matrix and a vector, or two matrices with the same number of rows:

```
> Aaugb := ⟨A|b⟩; #the augmented matrix
  AaugB := ⟨A|B⟩;
  Baugb := ⟨B|b⟩;
```

Multiply two matrices (or a matrix times a column vector):

```
> B.A; #we'd get an error if we tried A.B!
  B.b;
```

Matrix addition, scalar multiplication, integer powers, etc:

```
> C := Matrix(3, 3, [1, 0, 2, 2, 0, 1, 0, 0, 1]);
  2·C; #scalar multiplication
      #(shift-enter to add multiple line commands)
  (B + 2·C)^2; #what it looks like!
  B^-1; #inverse matrix if it exists
  B^-2; #the inverse squared
```

Solve a non–singular square matrix problem (non–singular means inversetransformation (and matrix) exist. We are solving Bx=b. Any one of the following commands will work!

```
> B^-1.b;
  ReducedRowEchelonForm(⟨B|b⟩);
  LinearSolve(B, b);
```

Solve any solvable matrix problem. Here we are solving Ax=b:

```
> LinearSolve(A, b); #Maple free parameters look funny!
  ReducedRowEchelonForm(⟨A|b⟩);
```