Math 2270−3
kernel and image − big example
September 16, 2009

Consider T(x)=Ax, for the large matrix

> *with*(*LinearAlgebra*) :

> *A* := *Matrix*(4, 6, [1, 2, 3, -1, -2, 6,
0, 1, 1, -2, -1, 3,
2, -4, -2, -1, 4, 3,
3, -2, 1, -2, 2, 9]);

$$A := \begin{bmatrix} 1 & 2 & 3 & -1 & -2 & 6 \\ 0 & 1 & 1 & -2 & -1 & 3 \\ 2 & -4 & -2 & -1 & 4 & 3 \\ 3 & -2 & 1 & -2 & 2 & 9 \end{bmatrix} \quad (1)$$

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1) Find an explicit representation for ker(T) (which we also write as ker(A)). Use this representation to find a basis for ker(T) and verify that it's a basis. You might want to use:

> *ReducedRowEchelonForm*(*A*);

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

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2a) For the same transformation, find an explicit representation for Im(T). Use the fewest number of vectors which still span this subspace. Verify that they are a basis. Hint: reuse your work in the previous part to cull vectors from this "column space." You can read this information from

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> ReducedRowEchelonForm(A);
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$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

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Notice column dependencies correspond to solutions to Ax=0, which do not change as you do elementary row operations. For example, since the third column of the reduced matrix is the sum of the first two, this must also be true for the original matrix!

2b) A clever way to get an especially nice basis for Im(T) is to use "elementary column operations" to replace the column vectors (which span Im(T)) with a nicer collection which still spans. This amounts to computing the reduced column echelon form of the matrix:

> $B := Transpose(A);$ *#turn columns into rows*

$$B := \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & -4 & -2 \\ 3 & 1 & -2 & 1 \\ -1 & -2 & -1 & -2 \\ -2 & -1 & 4 & 2 \\ 6 & 3 & 3 & 9 \end{bmatrix} \quad \textbf{(4)}$$

> $C := ReducedRowEchelonForm(B):$
> $Transpose(C);$ *#turn rows back into columns; this is the reduced column echelon form!*
> *#Identify a "nice" basis of* $Im(T)$. *Check work!*

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \textbf{(5)}$$

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