Math 2270–3
Second Exam Review Information
November 3, 2009

    We will use the usual problem session time and location, LCB 225, 10:45–12:15 Thursday Nov 5, for a problem session in which we will go over practice exam questions, or others you ask.

    The exam will cover section 3.4, 4.1–4.3, 5.1–5.4. In addition to being able to do the computations from these sections, you should know key definitions, the statements of the main theorems, and why they are true. The exam will be a mixture of computational and theoretical questions. As on the first exam, there will (also) be some true–false questions, see e.g. those at the ends of chapters 3, 4, 5.

One way to organize the topics is as follows:

Linear (combination) **Spaces, 3.4–4.3** (also called vector spaces)
    Definitions:
        Linear space
        subspace
        Linear transformation
        domain
        target
        kernel
        image
        rank
        nullity
        linear isomorphism
        linear combination
        span
        linear dependence, independence
        basis
        dimension
        coordinates with respect to a basis
        matrix of a linear transformation for a given basis
        change of coordinates matrix, as a special case of above
    Theorems:
        results about dimension: e.g. if dim(V)=n, then more than n vectors are ?, fewer than n vectors cannot ?, n linearly independent vectors automatically ?, n spanning vectors automatically are ?

      also, if a collection of vectors is dependent, it may be culled without decreasing the span; if a vector is not in the span of a collection of independent vectors, it may be added to the collection without destroying independence.
        the kernel and image of linear transformations are subspaces.
        rank plus nullity equals ?
        A linear transformation is an isomorphism if and only if ?
        Isomorphisms preserve ?
    Computations:
        Check if a set is a subspace
        Check if a transformation is linear
        Find kernel, image, rank, nullity of a linear transformation
        Check if a set is a basis; check spanning and independence questions.
        Find a basis for a subspace
        Find coordinates with respect to a basis
        Find the matrix of a linear transformation, with respect to a basis

Use the matrix of a linear transformation to understand kernel, image
Compute how the matrix of a linear trans changes if you change bases

Orthogonality (Chapter 5)
Definitions:
dot product in R^n
orthogonal
magnitude
unit vector
orthonormal collection
orthogonal complement to a subspace
orthogonal projection
angle
orthogonal transformation, orthogonal matrix
transpose
symmetric and antisymmetric matrices
least squares solutions to Ax=b
inner product spaces generalization (not on this exam, but maybe on final)
Theorems
dot product algebra
Pythagorean Theorem
Cauchy–Schwarz Inequality
Any basis can be replaced with an orthnormal basis (Gram Schmidt)
Algebra of the transpose operation
algebra of orthogonal matrices
Orthogonal complement geometry/algebra
Computations
coordinates when you have an orthonormal basis (in any inner product space)
Gram–Schmidt (works in any inner product space)
A=QR decomposition
orthogonal projections (works in any inner product space)
least squares solutions
application to best–line fit for data, or best polynomial fit, or power law fit
matrix for orthogonal projection