

Remark on change of basis matrices:

Let $\mathcal{A} = \{g_1, g_2, \dots, g_n\}$ be two bases for the linear space V .
 $\mathcal{B} = \{f_1, f_2, \dots, f_n\}$

Then $[c_1 g_1 + c_2 g_2 + \dots + c_n g_n]_{\mathcal{B}} = c_1 [g_1]_{\mathcal{B}} + c_2 [g_2]_{\mathcal{B}} + \dots + c_n [g_n]_{\mathcal{B}}$
 $= \begin{bmatrix} [g_1]_{\mathcal{B}} & [g_2]_{\mathcal{B}} & \dots & [g_n]_{\mathcal{B}} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$

So the change of coordinates matrix

$$S_{\mathcal{B} \leftarrow \mathcal{A}} = \begin{bmatrix} [g_1]_{\mathcal{B}} & [g_2]_{\mathcal{B}} & \dots & [g_n]_{\mathcal{B}} \end{bmatrix}; \quad \text{col}_j(S) = [g_j]_{\mathcal{B}}$$

Example $\mathcal{B} = \{1, x, x^2\}$, $\mathcal{A} = \{1+x, 1-x, x^2\}$ basis for \mathbb{P}_2
 f_1, f_2, f_3 g_1, g_2, g_3

$$S := S_{\mathcal{B} \leftarrow \mathcal{A}} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$[g_1]_{\mathcal{B}} \quad [g_2]_{\mathcal{B}} \quad [g_3]_{\mathcal{B}}$

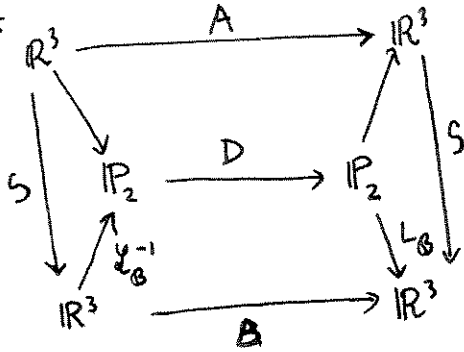
$$S^{-1} = S_{\mathcal{A} \leftarrow \mathcal{B}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$[f_1]_{\mathcal{A}} \quad [f_2]_{\mathcal{A}} \quad [f_3]_{\mathcal{A}}$

Example cont'd

$\mathcal{A} = \{1+x, 1-x, x^2\}$

$\mathcal{B} = \{1, x, x^2\}$



$$A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 1 \\ \frac{1}{2} & -\frac{1}{2} & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$[D(1+x)]_{\mathcal{A}} \quad [D(1-x)]_{\mathcal{A}} \quad [D(x^2)]_{\mathcal{A}}$

checked

$$SA = BS$$

i.e.

$$A = S^{-1}BS$$

(alternate way to get A from B, if direct column by column method is harder)

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$[D(1)]_{\mathcal{B}} \quad [D(x)]_{\mathcal{B}} \quad [D(x^2)]_{\mathcal{B}}$