Math 2270-3

- Exam review sheet (at end of class)
- Use Tuesday notes p. 3-5 as help for computer project, part B.
  (I'll be available for consultation on both parts.)

Before review, discuss det of 3×3 matrices, to lead into chapter 6; take abt half the lecture.

- Recall

$$\vec{v}\times\vec{w} := \begin{vmatrix} \hat{\imath} & v_1 & w_1 \\ \hat{\jmath} & v_2 & w_2 \\ \hat{k} & v_3 & w_3 \end{vmatrix} = \begin{bmatrix} \begin{vmatrix} v_2 & w_2 \\ v_3 & w_3 \end{vmatrix} \\ -\begin{vmatrix} v_1 & w_1 \\ v_3 & w_3 \end{vmatrix} \\ \begin{vmatrix} v_1 & w_1 \\ v_2 & w_2 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{bmatrix}$$

satisfies

$\vec{v}\times\vec{w} \perp \vec{v},\vec{w}$, $\vec{v}\times\vec{w} = -\vec{w}\times\vec{v}$

$\{\vec{v},\vec{w},\vec{v}\times\vec{w}\}$ right-handed

$\|\vec{v}\times\vec{w}\| = \|\vec{v}\|\|\vec{w}\|\sin\Theta$ $\quad \Theta = \measuredangle \vec{v},\vec{w}$

cross prod linear in each factor

these are "straightforward" algebra – you'll check in HW.

<u>Def</u> Write $A_{3\times3}$ in column form, $A = [\vec{u}|\vec{v}|\vec{w}]$

Then

$$|A| := \vec{u}\cdot(\vec{v}\times\vec{w})$$

If $\{\vec{v},\vec{w}\}$ are ind., they span a plane with normal vector $\vec{v}\times\vec{w}$

If $\vec{u}$ is <u>not</u> in this plane $\vec{u}\cdot(\vec{v}\times\vec{w})\neq 0$ (<u>iff</u>).

Therefore $\{\vec{u},\vec{v},\vec{w}\}$ are linearly independent iff $|A|\neq 0$

So

A^{-1} exists iff $|A|\neq 0$

(and there's a formula.

Interesting expansion patterns generalize $n=2$

$$\vec{u}\cdot(\vec{v}\times\vec{w}) = u_1v_2w_3 + u_2v_3w_1 + u_3v_1w_2$$
$$- u_1v_3w_2 - u_2v_1w_3 - u_3v_2w_1$$

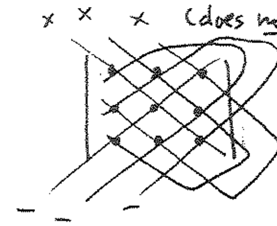
$\begin{bmatrix} \bullet & & \\ & \bullet & \\ & & \bullet \end{bmatrix}, \begin{bmatrix} & & \bullet \\ \bullet & & \\ & \bullet & \end{bmatrix}, \begin{bmatrix} & \bullet & \\ & & \bullet \\ \bullet & & \end{bmatrix}$ + patterns

$\begin{bmatrix} \bullet & & \\ & & \bullet \\ & \bullet & \end{bmatrix}, \begin{bmatrix} & \bullet & \\ \bullet & & \\ & & \bullet \end{bmatrix}, \begin{bmatrix} & & \bullet \\ & \bullet & \\ \bullet & & \end{bmatrix}$ – patterns

expansion contains all possible products using each column and row exactly once

the +,– coefficient depends on whether there are an even or odd # of "inversions" (pairs of pts where the ~~lower~~ later row entry has earlier col. entry)

3×3 ±'s: (does not generalize to n×n)



<u>example</u> $\begin{vmatrix} 1 & 2 & 0 \\ -1 & 3 & 6 \\ 2 & 4 & 1 \end{vmatrix} =$

<u>further algebra:</u>

- $|A| = \vec{u} \cdot (\vec{v} \times \vec{w})$ is linear in each column (keeping the other 2 columns fixed)
- If you interchange any two columns, you interchange the collections of + patterns & − patterns so sign of determinant changes
- $|A| = |A^T|$ because the + pattern collections & − pattern collections are preserved!
- so $|A|$ is linear in each row (keeping others fixed) and if you interchange rows $|\tilde{A}| = -|A|$.
- If you replace $row_i(A)$ with $row_i(A) + c\,row_k(A)$ $k \neq i$, det stays same!

$$i\text{th row} \rightarrow \begin{vmatrix} \vdots \\ row_k \\ row_i + c\,row_k \\ \vdots \end{vmatrix} = \underbrace{\begin{vmatrix} \vdots \\ row_k \\ c\,row_k \\ \vdots \end{vmatrix}}_{\substack{0 \\ \text{dep. rows!}}} + \underbrace{\begin{vmatrix} \vdots \\ row_k \\ row_i \\ \vdots \end{vmatrix}}_{\substack{\text{original} \\ \text{det}}}$$

<u>example</u> : use el. row ops to compute

$$\begin{vmatrix} 1 & 2 & 0 \\ -1 & 3 & 6 \\ 2 & 4 & 1 \end{vmatrix}$$