Math 2270-3
Monday Nov. 30

- key complex number facts from Wed Nov 25 notes

linear algebra with complex scalars ($s \in \mathbb{C}$ instead of $\mathbb{R}$)
Almost all concepts carry through
- complex vector space ; $\mathbb{C}^n$ vs. $\mathbb{R}^n$
- subspace
- span
- linear ind/dep
- basis
- row ops, rref
- matrix algebra, inverse
- dets

$\vdots$

evals, evects, eigenbasis (in $\mathbb{C}^n$), diagonalizability etc.

also, dot product on $\mathbb{C}^n$
$\vec{z}\cdot\vec{w} = \sum z_i w_i$
notice $\overline{\vec{z}\cdot\vec{w}} = \overline{\vec{z}}\cdot\overline{\vec{w}}$

<u>Example</u> Return to glucose-insulin discrete dynamical system. (8 Wed Maple handout)

$$\begin{bmatrix} G(t+1) \\ H(t+1) \end{bmatrix} = \begin{bmatrix} .9 & -.4 \\ .1 & .9 \end{bmatrix} \begin{bmatrix} G(t) \\ H(t) \end{bmatrix}$$

$$\text{IVP} \begin{cases} \vec{x}(t) = A^t \vec{x}(0) \\ \vec{x}(0) = \begin{bmatrix} 100 \\ 0 \end{bmatrix} \end{cases}$$

check Maple work:

$$|A - \lambda I| = \begin{vmatrix} .9-\lambda & -.4 \\ .1 & .9-\lambda \end{vmatrix} = (\lambda - .9)^2 + .04$$
$$= (\lambda - .9 - .2i)(\lambda - .9 + .2i)$$

roots $\lambda = .9 \pm .2i$

$\lambda = .9 + .2i$

$$\left[\begin{array}{cc|c} -.2i & -.4 & 0 \\ .1 & -.2i & 0 \end{array}\right]$$
$$\left[\begin{array}{cc|c} 1 & -2i & 0 \\ -2i & -4 & 0 \end{array}\right]$$
$$\left[\begin{array}{cc|c} 1 & -2i & 0 \\ 0 & 0 & 0 \end{array}\right] \quad 2iR_1 + R_2$$

$\vec{v} = \begin{bmatrix} 2i \\ 1 \end{bmatrix}$ $\longleftarrow$ for our IVP prefer $\boxed{\lambda = .9 + .2i \quad \vec{v} = \begin{bmatrix} -2 \\ i \end{bmatrix}}$

(could also use column dependency method)

$\longrightarrow$ $A\vec{v} = \lambda\vec{v} \Rightarrow \overline{A\vec{v}} = A\overline{\vec{v}}$ (A real)
$\overline{A\vec{v}} = \overline{\lambda\vec{v}} = \overline{\lambda}\,\overline{\vec{v}}$

conjugate eigenvalue has conjugate eigenspace

so for $\lambda = .9 - .2i$
$\vec{v} = \begin{bmatrix} -2i \\ 1 \end{bmatrix}$ works

for our IVP prefer

$\boxed{\lambda = .9 - .2i \quad \vec{v} = \begin{bmatrix} 2 \\ i \end{bmatrix}}$

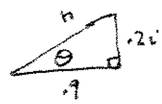
now let's solve glucose-insulin IVP
just like we did coyotee-roadrunner!

$$\begin{bmatrix}100\\0\end{bmatrix} = -25\begin{bmatrix}-2\\i\end{bmatrix} + 25\begin{bmatrix}2\\i\end{bmatrix}$$

so for $t \in \mathbb{N}$:
$$A^t\begin{bmatrix}100\\0\end{bmatrix} = -25\,(.9+.2i)^t\begin{bmatrix}-2\\i\end{bmatrix} + 25\,(.9-.2i)^t\begin{bmatrix}2\\i\end{bmatrix}$$

polar form:

$.9+.2i = re^{i\theta}$



$\theta = \arctan 2/9$

$r = \sqrt{.85}$

so
$$A^t\begin{bmatrix}100\\0\end{bmatrix} = -25\,\underset{\substack{\downarrow\\ r^t e^{i\theta t}}}{(re^{i\theta})^t}\begin{bmatrix}-2\\i\end{bmatrix} + 25\,\underset{\substack{\downarrow\\ r^t e^{-i\theta t}}}{(re^{-i\theta})^t}\begin{bmatrix}2\\i\end{bmatrix}$$

$$= 25\left[(.85)^{t/2}\right]\left[\left(\cos(t\theta) + i\sin(t\theta)\right)\begin{bmatrix}2\\-i\end{bmatrix} + \left(\cos(t\theta) - i\sin(t\theta)\right)\begin{bmatrix}2\\i\end{bmatrix}\right]$$

collect terms!

$$= 25\,\underbrace{(.85)^{t/2}\underbrace{\begin{bmatrix}4\cos(t\theta)\\2\sin(t\theta)\end{bmatrix}}_{\text{orbiting c.c. on ellipse } \frac{G^2}{16}+\frac{H^2}{4}=1}}_{\text{spiraling towards origin}}$$

See Maple picture!