Math 2270−3
Final Exam Review Information
December 11, 2009

Our final exam is this Monday morning, December 14th, 10:30−12:30 p.m., in our usual classroom LCB 215. I will let you stay until 1:00 p.m. There will be a review session tomorrow Saturday, from 11:00−1:00, also in LCB 215. Most of that session will be devoted to going over old exams, but please bring any other questions you may have − old exams never cover all possible question topics!

The exam will be comprehensive. Precisely, you can expect anything from chapters 1−8.2, and any supplementary material we also covered, such as the the affine transformation concepts related to fractals. Sections 8.3 and 5.5 (inner product spaces) will NOT be on the exam. In addition to being able to do computations, you should know key definitions, the statements of the main theorems, and why they are true. The exam will be a mixture of computational and theoretical questions. As on the midterms, there will be some true−false questions drawn from course material. And, as always, the exam is closed book and closed note.

Exam material will be weighted towards topics which have not yet been tested, i.e. chapter 6−8 material.

Copies of my final exams from the last two times I taught this course are posted on our web page, along with solutions. Of course, our exam will be somewhat different! It would also be a good idea to look over our midterms, class notes, homework problems, and the text. It always worked well for me to make my own course outline with the key ideas (which I would then make sure I could explain and work with), when I was in your shoes.

Matrix algebra in R^n. (Chapters 1−2) (implicitly used in all other topics)

- Linear systems
- and matrix equations Ax=b
  - intersecting hyperplane interpretation (linear system interpretation)
  - linear combination of columns interpretation
- rref(A|b) and rref(A): how to compute, how to use.
- matrix transformations T(x)=Ax, and affine transformations T(x)=Ax+b.
  - geometric properties (i.e. parallel lines get mapped to parallel lines, translations and scalings of any set are transformed into translations and scalings of the transformed set.)
- geometric transformations (scalings, rotations, projections, reflections, shears)
- inverse transformations and inverse matrices
- composition of transformations and matrix products
- matrix algebra (i.e. commutative, associative, distributive properties with addition and multiplication of matrices)

Linear Spaces, (Chapters 3−4) :Chapter 3 was about R^n, and in Chapter 4 we generalized these ideas to general linear (vector) spaces.

Definitions:

Linear space
subspace
Linear transformation
domain
target
kernel
image
rank
nullity
linear combination
span
linear dependence, independence
basis
dimension
linear isomorphism
coordinates with respect to a basis
matrix to change coordinates from one basis to another
matrix of a linear transformation for a given basis

Theorems:

results about dimension: e.g. if dim(V)=n, then more than n vectors are ?, fewer than n vectors cannot ?, n linearly independent vectors automatically ?, n spanning vectors automatically are ?

also, if a collection of vectors is dependent, it may be culled without decreasing the span; if a vector is not in the span of a collection of independent vectors, it may be added to the collection without destroying independence.

the kernel and image of linear transformations are subspaces.
rank plus nullity equals ?
A linear transformation is an isomorphism if and only if ?
Isomorphisms preserve ?

Computations:

Check if a set is a subspace (also, what are subspaces of R^n.)
Check if a transformation is linear
Find kernel, image, rank, nullity of a linear transformation
Check if a set is a basis; check spanning and independence questions.
Find a basis for a subspace
Find coordinates with respect to a basis
Find the matrix of a linear transformation, with respect to a basis
Use the matrix of a linear transformation to understand kernel, image
Compute how the matrix of a linear trans changes if you change bases
Similar matrices and their algebra.

Orthogonality (Chapter 5)

Definitions:

orthogonal
magnitude
unit vector
orthonormal collection
orthogonal complement to a subspace
orthogonal projection
angle via dot product
orthogonal transformation, orthogonal matrix
transpose
least squares solutions to Ax=b ; special case of fitting lines and polynomials to data.

Theorems

Pythagorean Theorem
Cauchy−Schwarz Inequality
Any basis can be replaced with an orthnormal basis (Gram Schmidt)
Algebra of the transpose operation
symmetric, antisymmetric
algebra of orthogonal matrices
Orthogonal complement of the orthogonal complement of V is V!

Computations

find coordinates when you have an orthonormal basis (in any inner product space)
Gram−Schmidt (in any inner product space)
A=QR decomposition
orthogonal projections (in any inner product space)
least squares solutions
application to best−line fit for data
find bases for the four fundamental subspaces of a matrix

Determinants (Chapter 6)

Definitions:

pattern−product definition of determinant

Theorems:

determinant of upper or lower triangular matrix.
determinant can be computed by expanding down any column or across any row
(You don’t need to know the proof of this theorem!)
determinant is linear in each fixed row or fixed column
effects of elementary row operations (or column ops) on the determinant of a matrix
area/volume of parallelepipeds and determinants
adjoint formula of the inverse, and Cramer’s rule
determinant of product is product of determinants
A is invertible if and only if its determinant is non−zero (iff rref(A)=I, iff ker(A)={0}, iff image (A)=R^n, etc.)

Computations:

determinants by row ops or original definition
inverse matrices via adjoint formula; Cramer’s rule for solving invertible systems.
computing areas or volumes of parallelepipeds.
the area or volume expansion factor of a linear transformation, the "orientation" of a basis of R^n.

Eigenvector concepts and applications (Chapters 7−8)

Definitions:

eigenvalue
eigenvector
characteristic polynomial
eigenspace
geometric and algebraic multiplicity of eigenvalues
eigenbasis for A
A is diagonalizable
all of the above definitions using complex scalars and vectors
Euler's formula for $e^{i\theta}$.
discrete dynamical system with transition matrix A
regular transition matrix
quadratic form
conic section, quadric surface

Theorems:

Similar matrices have identical characteristic polynomials (so same eigenvalues), and their eigenspaces with equal eigenvalues have the same dimension.
A is diagonalizable iff the geometric and algebraic multiplicities of each eigenvalue agree.
if A is n by n and has n distinct eigenvalues, then A is diagonalizable. (Holds in real and complex case.) In all other cases, see above!
$e^{i\alpha} e^{i\beta} = e^{i(\alpha+\beta)}$.
When is the zero vector a (asymptotically) stable equilibrium for a discrete dynamical system?
if A is real and symmetric then A has an orthonormal eigenbasis (Spectral Theorem)
any quadratic form can be diagonalized − i.e. there is an orthogonal change of coordinates so that in the new variables there are no cross terms.

Computations:

find characteristic polynomial, eigenvalues, eigenspace bases.
above, when eigenvalues are complex.
can you tell if two given matrices are similar or not similar?
find a closed form for the solution x(t) to a discrete dynamical system whose transition matrix A can be diagonalized, depending on x(0).
identify and graph a conic or quadric surface by finding the equation its coordinates satisfy in a rotated basis.