Math 2270-3
Tuesday Aug 25
§1.1-1.2

- on page 3 of yesterday's notes there's a lot of space to figure out the geometric configurations corresponding to the solution sets of 1, 2, or 3 (or more) equations in 3 unknowns. Let's do it!
- warm up on page 4 of today's notes

Here's the general set up for linear systems of equations:
You search for n-tuples (vectors) $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ for which LS is true:
(linear system)

$$LS \quad \begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

a_{ij}, coefficient in i^{th} eqtn of variable x_j, is <u>known</u>
b_i, right hand side (RHS) value, is <u>known</u>

$\{x_j\}_{j=1}^{n}$ are the n unknowns.

the problem of solving LS arises in applications throughout math, science, eng, econ., etc.
In this week's HW there are problems involving

- Leontief input-output models ~ originally for industrial demand in economics, actually related to e.g. google rank
- equilibrium temperature in a network (physics)
- polynomial interpolation of a function

we'll see lots more applications as course progresses.
there is an underlying theory for LS, and LS will also help us understand many other more advanced math topics.

Today: Gauss-Jordan elimination (did example Wed.)

greatest mathematician ever?

- algebra
- probability
- geometry

p. 19 story of how Gauss used (and invented) "least squares" solution to 17 linear eqns in ? unknowns, to calculate orbit of Ceres... (also p. 8, Chinese knew the algorithm for solving linear systems 2000 years earlier.).

Let's try solving (§1.2 #16)

$$3x_1 + 6x_2 + 9x_3 + 5x_4 + 25x_5 = 53$$
$$7x_1 + 14x_2 + 21x_3 + 9x_4 + 53x_5 = 105$$
$$-4x_1 - 8x_2 - 12x_3 + 5x_4 - 10x_5 = 11$$

by hand (?!) ~ to understand how to reduce matrices, since "reduced row echelon form", the result of performing G-J elimination, is not only practically useful, but also key to understanding many "why's".

We may use the 3 elementary row operations

- mult row by non-zero const
- interchange 2 rows
- replace a row by its sum with a multiple of another row.

$$\begin{array}{ccccc|c} 3 & 6 & 9 & 5 & 25 & 53 \\ 7 & 14 & 21 & 9 & 53 & 105 \\ -4 & -8 & -12 & 5 & -10 & 11 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 0 & 5 & 6 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

rref

Reduced row echelon form: p.16

a) If a row has any non-zero entries, then the first non-zero entry is a 1, called the leading 1 of this row

b) If a column contains a (row's) leading 1, all other column entries are zero

c) as you move down the rows of the matrix the leading 1's move to the right by at least one column

does each matrix have exactly one rref?
ans: yes; easier to show in chapter 3.

Solution to original system:

What if rref had been

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 0 & 5 & 6 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad ?$$

Maple Check!

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> with(LinearAlgebra) :
> A := [  3   6   9  5  25   53
          7  14  21  9  53  105
         -4  -8 -12  5 -10   11 ] :
> ReducedRowEchelonForm(A);
```

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 5 & 6 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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>
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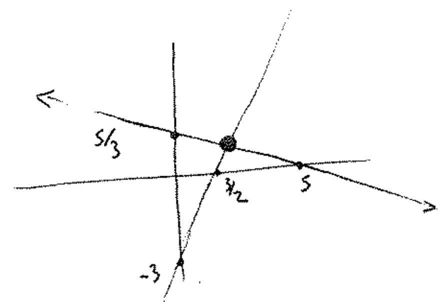
We talked yesterday about why elementary row operations, which correspond to elementary equation operations, do not change the solution set of LS. (I'll assign HW problems next week that will make you write down an explanation.

Here's a nice small example of this – although you may change the solution set to individual equations in LS (since you're changing individual equations), you won't change the solution set to the whole linear system LS:

<u>geometric example</u>

LS $\quad \begin{aligned} x + 3y &= 5 \\ 2x - y &= 3 \end{aligned}$

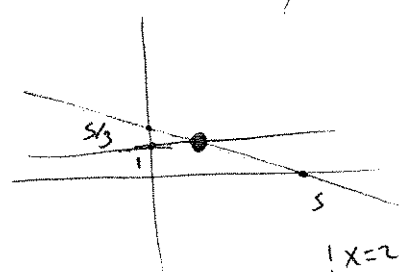
$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right]$$



$2E_1 + E_2$ then $\frac{E_2}{-7}$ $\quad \begin{aligned} x + 3y &= 5 \\ -7y &= -7 \end{aligned}$

$-2R_1 + R_2$ then $\frac{R_2}{-7}$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -7 & -7 \end{array}\right]$$

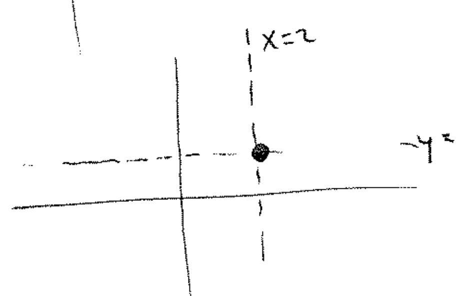


$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 1 & 1 \end{array}\right]$$

$3E_2 + E_1$ $\quad \begin{aligned} x &= 2 \\ y &= 1 \end{aligned}$

$-3R_2 + R_1$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$



the analogous geometry, with planes in $\mathbb{R}^3$ instead of lines in $\mathbb{R}^2$, was occurring in yesterday's example.