Fractal Example
Math 2270-1
September 14, 2005

This document is written using Maple. It is an example of how might do part B #2, assuming you hadn't found a template for the "gothic tree" (below) anywhere and were making it up....

Step 1: I opened the maple file
http://www.math.utah.edu/~fractals/Lpictures.mws
and executed the worksheet. This loaded the TESTMAP and AFFINE1 procedures which I shall use to define the transformations and make the L-picture diagrams.

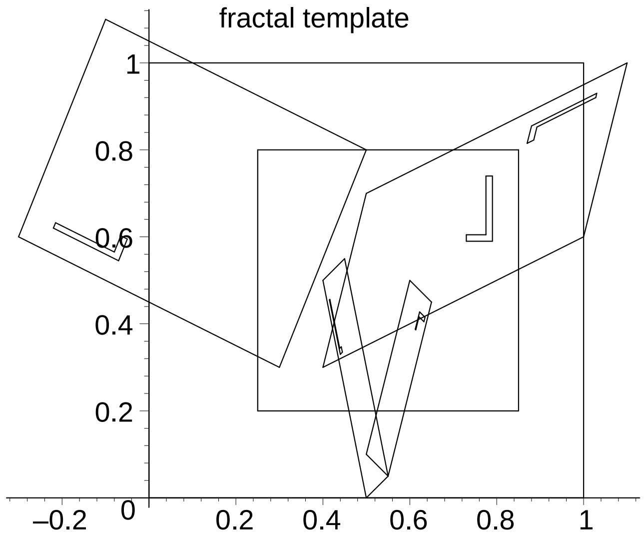
Step 2: Stealing and modifying commands from the file
http://www.math.utah.edu/~fractals/Sierpinski.mws
I created a picture five affine transformations which, it seemed to me, would generate a tree-like fractal, and then encoded them using AFFINE1

```
> f1:=P->AFFINE1(P,-.6,0,0,.6,.85,.2);
  f2:=P->AFFINE1(P,.2,.5,-.6,.3,.3,.3);
  f3:=P->AFFINE1(P,-.1,-.4,.6,.3,.5,.7);
  f4:=P->AFFINE1(P,-.1,-.4,.05,-.05,.6,.5);
  f5:=P->AFFINE1(P,.05,.05,-.1,.5,.5,0);
```

$$f1 := P \to \mathrm{AFFINE1}(P, -0.6, 0, 0, 0.6, 0.85, 0.2)$$
$$f2 := P \to \mathrm{AFFINE1}(P, 0.2, 0.5, -0.6, 0.3, 0.3, 0.3)$$
$$f3 := P \to \mathrm{AFFINE1}(P, -0.1, -0.4, 0.6, 0.3, 0.5, 0.7)$$
$$f4 := P \to \mathrm{AFFINE1}(P, -0.1, -0.4, 0.05, -0.05, 0.6, 0.5)$$
$$f5 := P \to \mathrm{AFFINE1}(P, 0.05, 0.05, -0.1, 0.5, 0.5, 0)$$

Now I tested the transformations with TESTMAP:

```
> TESTMAP([f1,f2,f3,f4,f5]);
```

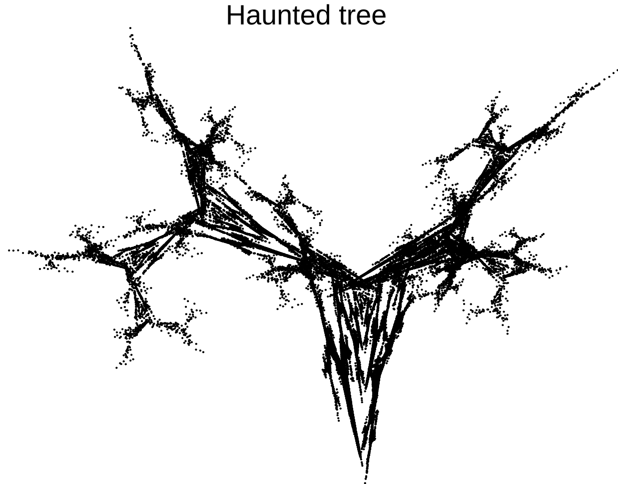


Let’s see what fractal this generates!

```
> S:={[0,0]}:#initial set consisting of one point
  5^7;  #want less than 200,000 points
```

78125

```
> for i from 1 to 7 do
  S1:=map(f1,S);
  S2:=map(f2,S);
  S3:=map(f3,S);
  S4:=map(f4,S);
  S5:=map(f5,S);
  S:=‘union‘(S1,S2,S3,S4,S5);
  od:
> pointplot(S,symbol=point,scaling=constrained,
      axes=none,title=‘Haunted tree‘);
```



Note on contractions! In order for the theory we've talking about to apply, each transformation must be a contraction of the plane. There is actually a computation you can do to check whether you're O.K. If A is the matrix of your transformation function and transpose(A) is the transposition of it which interchanges rows and columns, then the eigenvalues of transpose(A) times A are the squares of the maximum and minimum stretching (which varies according to direction) - you want the larger of these numbers to be less than one! For example, the matrix of the left-most box above is

```
> with(linalg):
  A:=matrix(2,2,[.2,-.06,.5,.3]);
  eigenvals(transpose(A)&*A);
Warning, the protected names norm and trace have been redefined and unprotected
```

$$A := \begin{bmatrix} 0.2 & -0.06 \\ 0.5 & 0.3 \end{bmatrix}$$

$$0.02243, 0.3612$$

```
> sqrt(.3612); #maximum stretch factor for A
```

$$0.6010$$

We'll understand the math which I just claimed, by the end of the course!