

Math 2270-1

Fri 9/9

NOTE: MONDAY CLASS WILL BE HELD IN MATH COMPUTER CLASSROOM LCB 115

We will begin Maple project, which you will hand in 2 weeks from today: 9/23

HW for Fri 9/16

(1)

2.3 $(29, 33, 34, 38)$

2.4 $(16, 17, 19)$ 20, 21, (23) 25

$(28, 43, 44, 86)$

p. 97-99: Explain!

3, $(6, 9)$ 12 (15) 18 $(21, 24)$ 27

$(30, 33, 36)$

Inverse transformations & matrices

Recall from Wed. that if $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a matrix (linear) transformation, $f(\vec{x}) = A\vec{x}$

then the inverse function can exist if and only if $n=m$ AND $\text{rref}(A) = I_{n \times n}$

also, if $f^{-1}(\vec{y})$ exists, it is a matrix transformation $f^{-1}(\vec{y}) = B\vec{y}$ (call $B=A^{-1}$).

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

In this case $f^{-1}(f(\vec{x})) = \vec{x} \quad \forall \vec{x} \in \mathbb{R}^n$

we carefully checked that matrix of composition is product matrix

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$$B(A\vec{x}) = \vec{x}$$

$$(BA)\vec{x} = \vec{x}$$

in particular

$$BA \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\uparrow \vec{e}_i$$

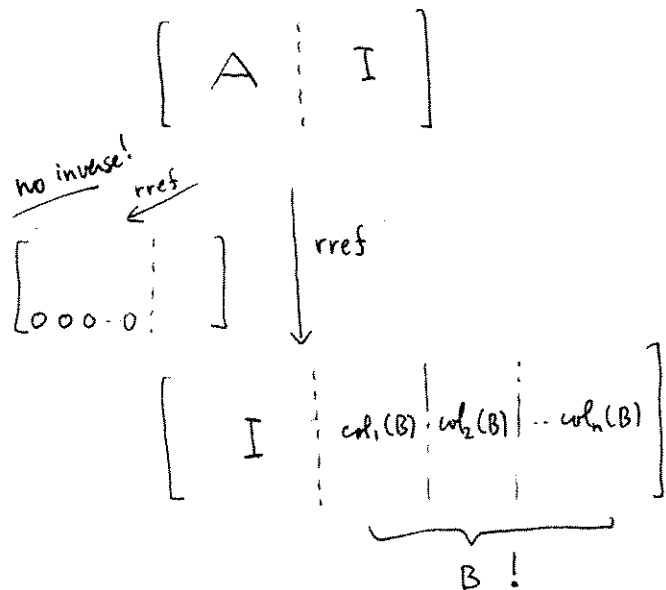
$$\text{col}_i(BA) = \vec{e}_i$$

also $\text{col}_j(BA) = \vec{e}_j = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$ ← j^{th} entry

thus $BA = I$.
Similarly $AB = I$.

So to find $B = A^{-1}$ we solve $AB = I$ synthetically

$$A \begin{bmatrix} | & | & & | \\ \text{col}_1(B) & \text{col}_2(B) & \dots & \text{col}_n(B) \\ | & | & & | \end{bmatrix} = \begin{bmatrix} | & | & & | \\ 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$



this can also be understood in terms of the inverse transformation:

$$\begin{bmatrix} | & | \\ A & I \end{bmatrix}$$

stands for $A\vec{x} = \vec{y}$:

$$a_{11}x_1 + a_{12}x_2 + \dots = y_1$$

$$a_{21}x_1 + \dots = y_2$$

...

$$a_{n1}x_1 + \dots + a_{nn}x_n = y_n$$

If $\text{rref}(A) = I$, we end up with

$$x_1 = b_{11}y_1 + b_{12}y_2 + \dots$$

$$x_2 = b_{21}y_1 + b_{22}y_2 + \dots$$

$$\dots$$

$$x_n = b_{n1}y_1 + \dots$$

$$\boxed{\vec{x} = B\vec{y}} !$$

eg. Find A^{-1} for $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ (today Hw).

$$\text{ans: } A^{-1} = \begin{bmatrix} 0 & -\frac{1}{2} & \frac{3}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$