Math 2270-1
Fri 9/9

NOTE: MONDAY CLASS WILL BE HELD IN MATH COMPUTER CLASSROOM <u>LCB 115</u>

We will begin Maple project, which you will hand in 2 weeks from today: 9/23

HW for Fri 9/16

2.3 29, 33, 34, 38
2.4 16, 17, 19 20, 21, 23 25
28, 43, 44, 86
p. 97-99 : Explain!
3, 6, 9 12 15 18 21, 24 27
30, 33, 36



## Inverse transformations & matrices

Recall from Wed. that if $f: \mathbb{R}^n \to \mathbb{R}^m$ is a matrix (linear) transformation,
$$f(\vec{x}) = A\vec{x}$$
then the inverse function can exist if and only if $n=m$ AND $\operatorname{rref}(A) = I_{n\times n} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$

also, if $f^{-1}(\vec{y})$ exists, it is a matrix transformation
$$f^{-1}(\vec{y}) = B\vec{y} \qquad \text{(call } B = A^{-1}\text{)}.$$

In this case $f^{-1}(f(\vec{x})) = \vec{x} \quad \forall \vec{x} \in \mathbb{R}^n$

we carefully checked that matrix of composition is product matrix

$$B(A\vec{x}) = \vec{x}$$
$$(BA)\vec{x} = \vec{x}$$

in particular
$$\underbrace{BA \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\operatorname{col}_1(BA) = \vec{e}_1} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$
$\uparrow$ $\vec{e}_1$

also $\operatorname{col}_j(BA) = \vec{e}_j = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ 0 \end{bmatrix} \leftarrow j^{th}$ entry

thus $BA = I$.
Similarly $AB = I$.

So to find $B = A^{-1}$ we solve $AB = I$ synthetically

$$A \left[ \begin{array}{c|c|c} col_1(B) & col_2(B) & \cdots col_n(B) \end{array} \right] = \left[ \begin{array}{c|c|c|c} 1 & 0 & & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & 0 & & \vdots \\ & \vdots & & 0 \\ 0 & 0 & & 1 \end{array} \right]$$

$$\left[ \begin{array}{c|c} A & I \end{array} \right]$$

no inverse! $\xleftarrow{rref}$

$$\left[ \begin{array}{c|c} & \\ 0\,0\,0 \cdots 0 & \end{array} \right]$$

$\Big\downarrow$ rref

$$\left[ \begin{array}{c|c|c|c} I & col_1(B) & col_2(B) & \cdots col_n(B) \end{array} \right]$$

$\underbrace{\qquad\qquad\qquad}_{B\,!}$

e.g. Find $A^{-1}$ for
$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$
(today HW).

this can also be understood in terms of the inverse transformation:

$$\begin{array}{cc} x_1\ x_2 \cdots & y_1 \cdots y_n \\ \left[ \begin{array}{c|c} A & I \end{array} \right. & \left. \right] \end{array}$$

stands for $\boxed{A\vec{x} = \vec{y}}$:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots &= y_1 \\ a_{21}x_1 + \cdots &= y_2 \\ &\vdots \\ a_{n1}x_1 + \cdots + a_{nn}x_n &= y_n. \end{aligned}$$

If $rref(A) = I$, we end up with

$$\begin{aligned} x_1 &= b_{11}y_1 + b_{12}y_2 + \cdots \\ x_2 &= b_{21}y_1 + b_{22}y_2 + \cdots \\ &\ddots \\ x_n &= b_{n1}y_1 + \cdots \end{aligned}$$

$\boxed{\vec{x} = B\vec{y}}$ !

ans: $A^{-1} = \begin{bmatrix} 0 & -1/2 & 3/2 \\ 0 & 1/2 & -1/2 \\ 1 & -1/2 & -1/2 \end{bmatrix}$