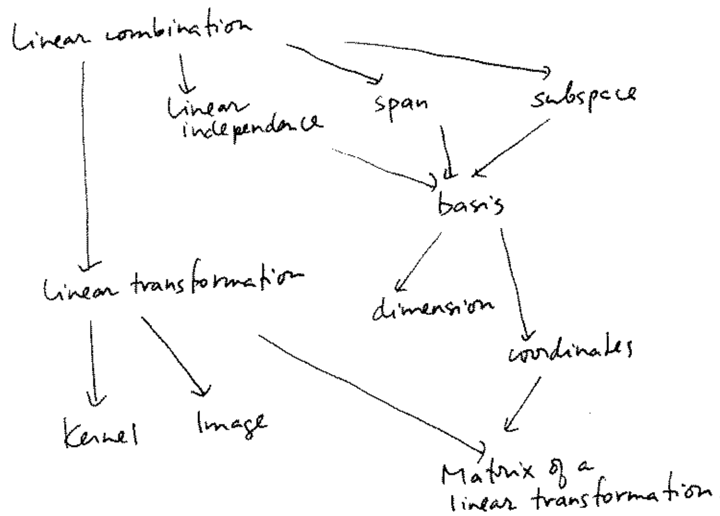
Math 2270
Fri 30 Sept.

HW due in 2 weeks, 10/14

4.1 1, 2, 5, 6, 10, 13, 14, 20, 25, 30, 35, 48
4.2 1, 2, 5, 6, 10, 19, 26, 27
4.3 1, 2, 3, 5, 7, 8, 13, 15, 21, 26, 29, 35, 42, 49

§ 4.1 Linear Spaces
(called vector spaces in most books)

In chapter 3 we followed this tree of ideas for $\mathbb{R}^n$: (p. 155 text)




A linear space (or vector space) is a collection of objects for which you can take linear combinations, and so that the expected algebraic properties (below) hold. (p. 153 text).
it turns out there are many more of these spaces than $\mathbb{R}^n$!

Def: A linear space $V$ is a set endowed with a rule for addition (If $f, g \in V$ then so is $f+g$) and a rule for scalar multiplication (If $f \in V$ and $k \in \mathbb{R}$ then $kf \in V$), such that these operations satisfy the following 8 rules ($\forall\, f, g, h \in V$, $\forall\, c, k \in \mathbb{R}$)

1. $(f+g)+h = f+(g+h)$
2. $f+g = g+f$
3. $\exists$ neutral element $n \in V$ s.t. $f+n = f$ $\forall f \in V$. This $n$ is unique and is denoted by $0$
4. $\forall f \in V\ \exists\, g \in V$ s.t. $f+g=0$. This $g$ is unique and is denoted by $-f$
5. $k(f+g) = kf + kg$
6. $(c+k)f = cf + kf$
7. $c(kf) = (ck)f$
8. $1f = f$.

Lots of examples!
See text p. 154–161.

<u>Example 3</u> $F(\mathbb{R}, \mathbb{R}) = \{ f : \mathbb{R} \to \mathbb{R} \}$

$$(f+g)(x) := f(x) + g(x)$$
$$(kf)(x) := k \cdot f(x)$$

Verify properties!
What's the neutral element?

<u>Example 4</u> $\mathbb{R}^{m\times n} := \{ M_{m\times n}$, i.e. all $m\times n$ matrices$\}$.
addition & scalar multiplication of matrices

<u>Example 5</u> The set of all infinite sequences of real numbers

$$(x_0, x_1, x_2, x_3, \ldots) + (y_0, y_1, y_2, \ldots) := (x_0+y_0,\ x_1+y_1,\ x_2+y_2, \ldots)$$
$$k(x_0, x_1, x_2, \ldots) := (kx_0, kx_1, kx_2, \ldots)$$

Example 8 $\mathbb{C} = \{ a + ib \text{ s.t. } a, b \in \mathbb{R} \}$. $(i = \sqrt{-1})$, $i^2 = -1$

$(a+ib) + (c+id) := (a+c) + i(b+d)$

$k(a+ib) := ka + ikb \qquad k \in \mathbb{R}$

For a linear space $V$, can you define

- linear combination of $f_1, \ldots f_n$:

- a subspace $W$ (and notice $W$ itself is a linear space!)

- span $\{f_1, \ldots f_n\}$

- $\{f_1, \ldots f_n\}$ linearly independent / linearly dependent

- $\{f_1, \ldots f_n\}$ are a basis for $V$:
  coords for $v \in V$ w.r.t basis $\mathcal{B} = \{f_1, \ldots f_n\}$.

- $\dim V$

[redacted]

Example $\cancel{} \ \mathbb{P}_2 := \{ p(x) = a_0 + a_1 x + a_2 x^2 : a_i \in \mathbb{R} \} \subset \mathbb{F}(\mathbb{R}, \mathbb{R})$

Show $\mathbb{P}_2$ is a subspace of $\mathbb{F}(\mathbb{R}, \mathbb{R})$

Find a "natural" basis.

Is $\{1+x, 1+x^2, x+x^2\}$ another basis?

Find coords of $p(x) = x^2$ with respect to both bases