Math 2270-1
Friday Sept 23

HW for Fri 9/30
3.4 1, 3, 5, 6, 11, 14, 15, 16, 17, 21, 22, 31, 33, 43, 53, 56

- We will go over the review sheet & practice exam on Monday. (Solutions to practice exam are posted on our home page!)
- Finish Wednesday notes ~ pages 11 & 12

On page 5 (Tuesday) we showed that any finite set of vectors spanning a subspace W can be successively culled of dependent vectors, and ultimately replaced with a linearly independent set having the same span, i.e. W.

An analogous fact is

<u>Theorem</u>: If $\{\vec{v}_1, .. \vec{v}_k\} \subset W$ is a linearly independent collection of vectors, and $\dim(W) = n > k$, then there is a basis for W consisting of $\{\vec{v}_1, .. \vec{v}_k, \vec{w}_{k+1}, .. \vec{w}_n\}$

<u>proof</u>: Since $\vec{v}_1, .. \vec{v}_k$ don't span W $(k<n)$, pick any vector not in span$\{\vec{v}_1, .. \vec{v}_k\}$, call it $\vec{w}_{k+1}$.

Then $\{\vec{v}_1, .. \vec{v}_k, \vec{w}_{k+1}\}$ is still linearly independent!

Because, if

$$c_1\vec{v}_1 + .. + c_k\vec{v}_k + c_{k+1}\vec{w}_{k+1} = \vec{0}$$

$c_{k+1} = 0$

$\Rightarrow c_1\vec{v}_1 + .. + c_k\vec{v}_k = \vec{0}$

$\Rightarrow c_1 = c_2 = \ldots = c_k = 0$

by l.i. of original set

$c_{k+1} \neq 0$

$\Rightarrow \vec{w}_{k+1} = -\frac{c_1}{c_{k+1}}\vec{v}_1 - \frac{c_2}{c_{k+1}}\vec{v}_k - \ldots - \frac{c_k}{c_{k+1}}\vec{v}_k$

(contradiction!)
So this case didn't happen.

Continue this process successively, won't get a spanning set before finding $\{\vec{v}_1, .. \vec{v}_k, \vec{w}_{k+1}, .. \vec{w}_n\}$ $\boxed{\text{WHY?}}$

and at this point the collection must span, since otherwise we could append $\vec{w}_{n+1}$, creating $(n+1)$ independent vectors in an n-dim'l space!

Note on "proof by contradiction"

$p \Rightarrow q$ is logically equivalent to not $q \Rightarrow$ not p.

"statement p implies statement q"
or "if p is true then q is true"

<u>example</u>: If your last name starts with the letter Z, then you ~~will~~ get an A on exam 1 is logically equivalent to If you didn't get an A on exam 1, your last name didn't start with Z!

Why? The assumption is that each of p or q can (only) be either true or false. Make a truth table of all 4 possibilities about the truth of (p, q):

(p,q)	q	
p	(T,T)	(T,F)
	(F,T)	(F,F)

$p \Rightarrow q$ means the possibility $\overset{p\ q}{(T,F)}$ cannot occur, but the other three can!

not $q \Rightarrow$ not p eliminates exactly the same $\overset{p\ q}{(T,F)}$, although now you're focusing on the 2^{nd} column of the table rather than the 1^{st} row!

<u>Theorem</u>: If $\dim(W) = k$, then

(a) any k independent vectors in W automatically span! So are a basis

Why? use pages 13, 11

(b) any k vectors which span W are automatically independent! So are a basis

Why? use pages 13, 11

<u>examples</u>: Are $\left\{ \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix} \right\}$ a basis for the plane $x + 2y + 3z = 0$?

Find a basis for $\mathbb{R}^3$ including $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$