Math 2270-1
9/21 Wednesday
Chapter 3 cont'd

We have yet to do the neat example on page 8, Tuesday.
Here's some extra space:

Here's a neat fact, can you explain it ??

- When you do row operations to a matrix, column dependencies stay the same!
  For example, in the page 8 example

$$\mathrm{col}_5(\mathrm{rref}(A)) = -\mathrm{col}_1(\mathrm{rref}(A)) + 4\,\mathrm{col}_3(\mathrm{rref}(A))$$

so

$$\mathrm{col}_5(A) = -\mathrm{col}_1(A) + 4\,\mathrm{col}_3(A) \quad !$$

Prove! Use rref!!

1. Every basis of $\mathbb{R}^n$ must have exactly n vectors
   - more than n vectors must be dependent:

e.g. could $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \end{bmatrix} \right\}$ be linearly ind?

- fewer than n vectors cannot span:

e.g. could $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 27 \\ 36 \\ 2 \end{bmatrix} \right\}$ ~~be~~ span $\mathbb{R}^3$?

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2. $\{\vec{v}_1, \vec{v}_2, \dots \vec{v}_n\}$ is a basis of $\mathbb{R}^n$ iff $\text{rref}\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \end{bmatrix} = I$.

3) <u>Theorem</u>: Let $\{\vec{v}_1, \vec{v}_2, .. \vec{v}_k\}$ be a basis for the subspace $W \subset \mathbb{R}^n$.
Then <u>every</u> basis of W has exactly k vectors $\longrightarrow$ and so <u>dimension</u> of W, $= k$, is well-defined.

(a) If $\{\vec{w}_1, \vec{w}_2, .. \vec{w}_\ell\} \subset W$ with $\ell > k$, then this set is dependent

(b) If $\{\vec{w}_1, \vec{w}_2, .. \vec{w}_\ell\} \subset W$ with $\ell < k$, then this set cannot span W
(because if the w_j's DO span, the $\vec{v}_i$'s would have to be dependent!)

4 (rank + nullity Theorem)

If $f(\vec{x}) = A\vec{x}$

$f : \mathbb{R}^n \to \mathbb{R}^m$

$$\underbrace{\dim(\ker f)}_{\text{"nullity"}} + \underbrace{\dim(\text{image}(f))}_{\text{"rank"}} = \dim(\text{domain}) = n$$