

Math 2270-1
Fri 2 Sept.

• Finish Wed. notes

HW for Fri Sept 1
 • Bob problems on today's notes
 2.1 1, 3, 4, 5, 6, 7, 8, 10, 13, 16, 17, 19, 21, 24, 25
 2.2 1, 2, 7, 9, 10, 11, 12, 13, 17, 27, 35, 36, 37, 38, 43, 44, 49
 2.3 1, 2, 5, 6, 2.4 1, 4, 14

• A generalization of "linear" (matrix) transformations allows you to translate by a fixed vector after performing the matrix product:

$$f(\vec{x}) = A\vec{x} + \vec{b}$$

Affine transformations are the backbone of single and multivariable Calculus, because every differentiable function is almost affine on small scales!

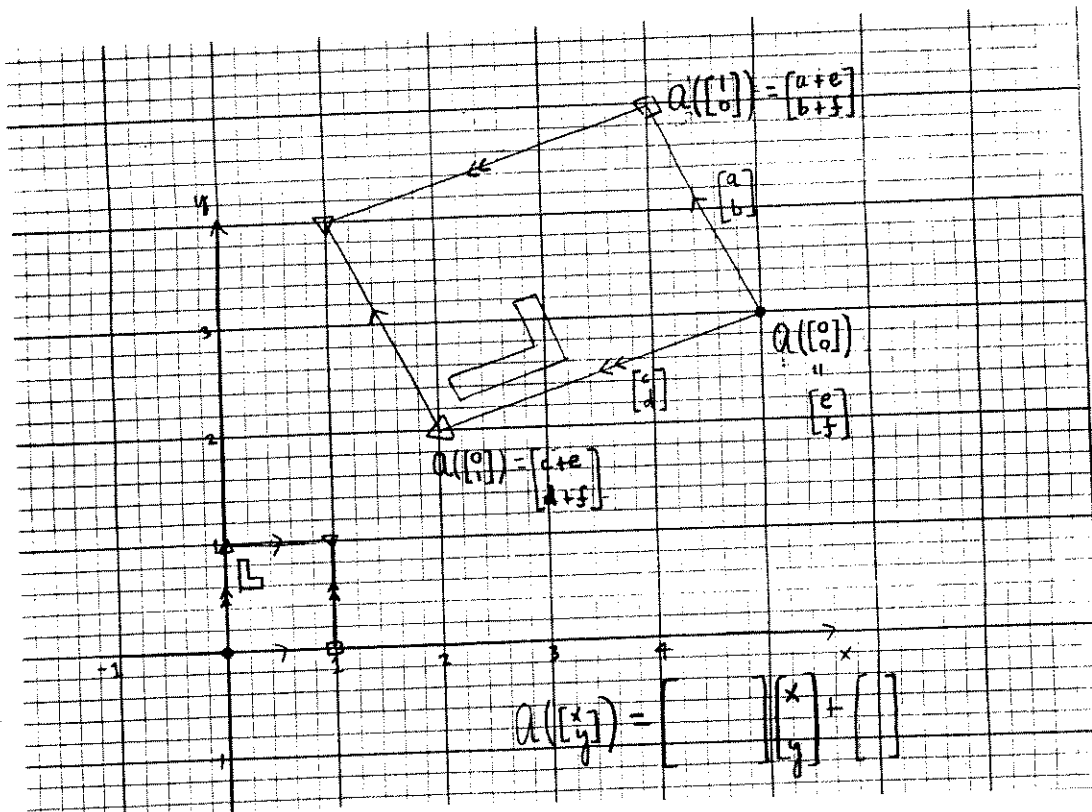
The properties of matrix transformations (p. 5 Wed) yield the same geometric properties for affine ones (eg. parallel lines \rightarrow parallel lines, midpoints \rightarrow midpoints, scaled sets \rightarrow scaled image sets.)

Affine transformation template

$$Q\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ax+cy+e \\ bx+dy+f \end{bmatrix} = x \begin{bmatrix} a \\ b \end{bmatrix} + y \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

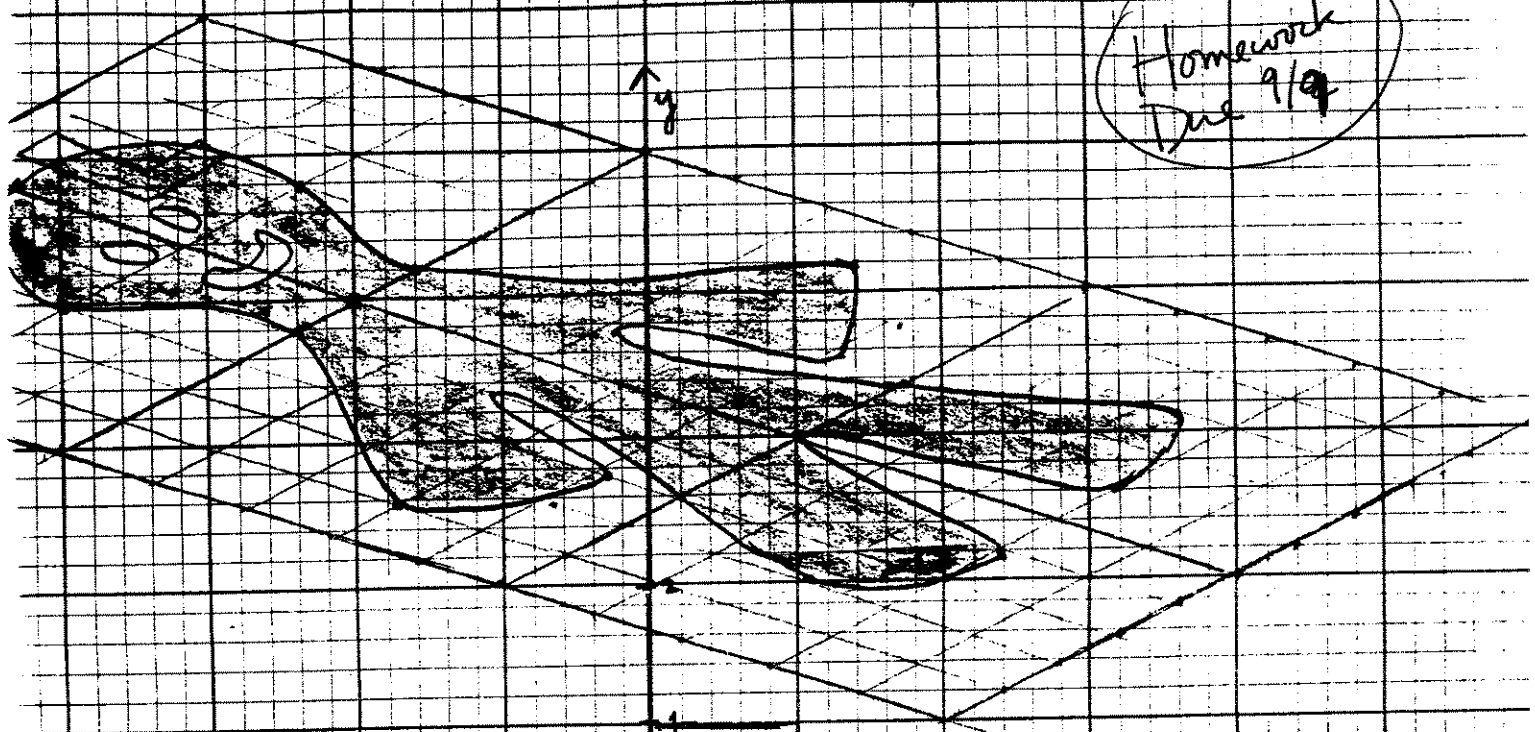
this part distorts Bob

this part translates him



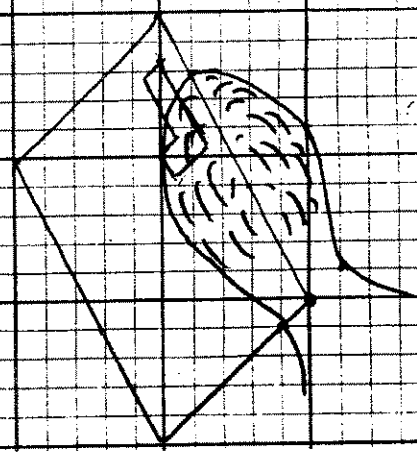
Bob & L-box transform themselves

Homework
Due 9/9



- ① Find formulas for the two affine maps which are shown
- ② Show where the L-box is transformed to by

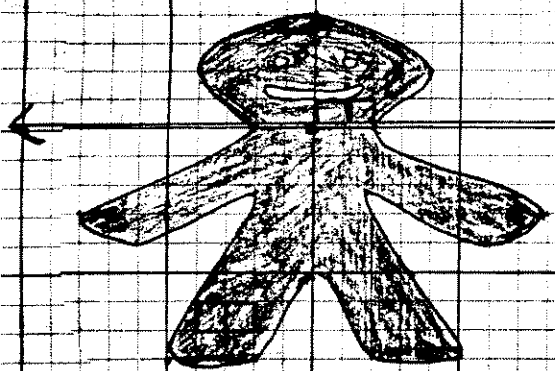
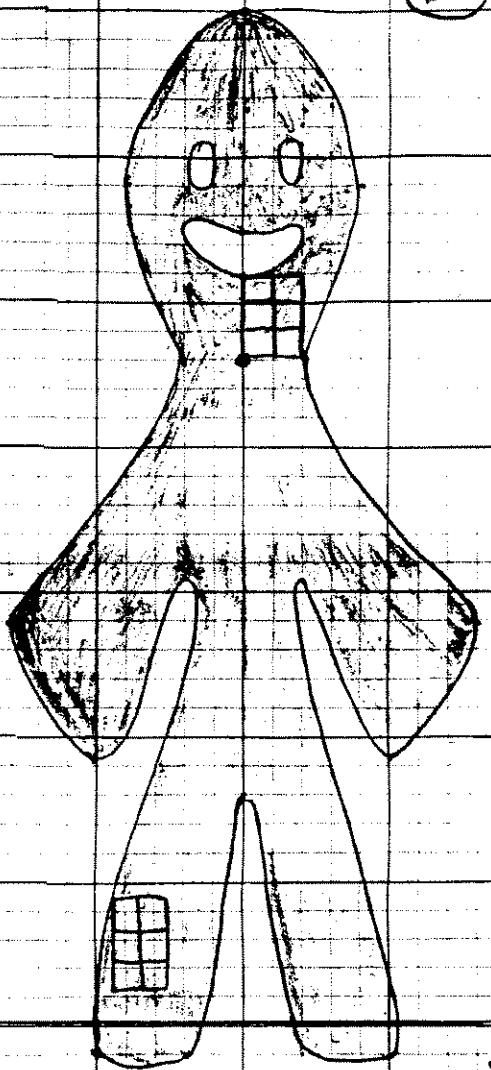
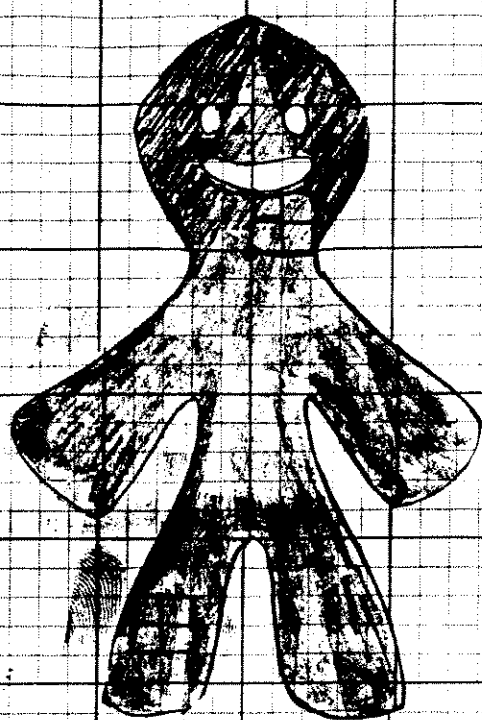
$$A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \end{pmatrix}$$



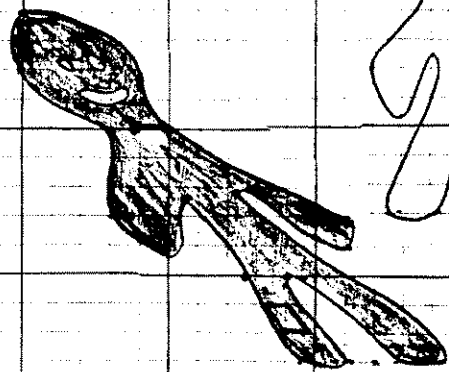
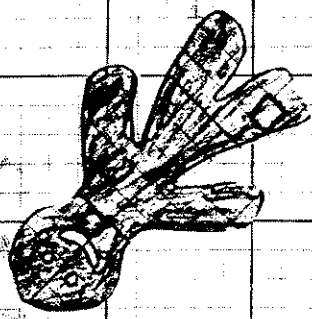
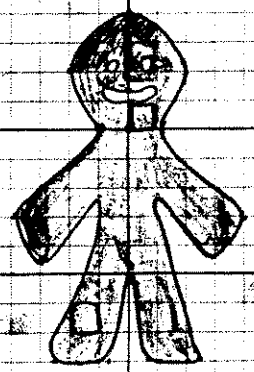
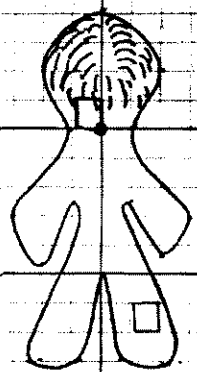
y

Bob transforms himself.

WHAT FORMULAS?



Bob



z

1

1

x

easy transformations:

translation:

$$A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \\ \end{bmatrix}$$

scaling and translation:

$$A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x \\ 5y \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \\ \end{bmatrix}$$

reflection: (across y-axis)

$$A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -x \\ y \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

etc.

also rotations

rotate by θ radians
counterclockwise:

$$A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

also (later) reflections
through arbitrary lines...

