Math 2270-1
Second Exam Review Information
November 2, 2005

We will use the usual problem session time and location, LCB 121, 9:40-10:30 Thursday Nov 2, for a problem session in which we will go over the practice exam.

The exam will cover section 3.4, 4.1-4.3, 5.1-5.5. In addition to being able to do the computations from these sections, you should know key definitions, the statements of the main theorems, and why they are true. The exam will be a mixture of computational and theoretical questions. As on the first exam, there will (also) be some true-false questions, see e.g. those at the ends of chapters 3, 4, 5.

One way to organize the topics is as follows:

Linear (combination) **Spaces, 3.4-4.3** (also called vector spaces)

Definitions:

Linear space
subspace
Linear transformation
domain
codomain
kernel
image
rank
nullity
linear isomorphism
linear combination
span
linear dependence, independence
basis
dimension
coordinates with respect to a basis
matrix of a linear transformation for a given basis

Theorems:

results about dimension: e.g. if dim(V)=n, then more than n vectors are ?, fewer than n vectors cannot ?, n linearly independent vectors automatically ?, n spanning vectors automatically are ?

also, if a collection of vectors is dependent, it may be culled without decreasing the span; if a vector is not in the span of a collection of independent vectors, it may be added to the collection without destroying independence.

the kernel and image of linear transformations are subspaces.
rank plus nullity equals ?
A linear transformation is an isomorphism if and only if ?
Isomorphisms preserve ?

Computations:

Check if a set is a subspace
Check if a transformation is linear
Find kernel, image, rank, nullity of a linear transformation
Check if a set is a basis; check spanning and independence questions.

Find a basis for a subspace
Find coordinates with respect to a basis
Find the matrix of a linear transformation, with respect to a basis
Use the matrix of a linear transformation to understand kernel, image
Compute how the matrix of a linear trans changes if you change bases

Orthogonality (Chapter 5)

Definitions:

orthogonal
magnitude
unit vector
orthonormal collection
orthogonal complement to a subspace
orthogonal projection
angle
correlation coefficient (not on exam, but interesting)
orthogonal transformation, orthogonal matrix
transpose
least squares solutions to Ax=b
inner product spaces

Theorems

Pythagorean Theorem
Cauchy-Schwarz Inequality
Any basis can be replaced with an orthnormal basis (Gram Schmidt)
Algebra of the transpose operation
symmetric, antisymmetric
algebra of orthogonal matrices
Orthogonal complement of the orthogonal complement of V is V!

Computations

coordinates when you have an orthonormal basis (in any inner product space)
Gram-Schmidt (in any inner product space)
A=QR decomposition
orthogonal projections (in any inner product space)
least squares solutions
application to best-line fit for data
matrix for orthogonal projection
The four fundamental subspaces of a matrix