Math 2270-1
Project 2, part A

A Power Law For Human Heights and Weights

Solutions

Here is the height-weight data which the class collected. Thanks to all who contributed! Heights vary from baby (18 inches) to tall person (75 inches). Weights vary from 6 to 250 pounds. .

```
> restart:with(linalg):with(plots):
Warning, the protected names norm and trace have been redefined and unprotected

Warning, the name changecoords has been redefined
```

```
> S:=[[62.5,107],  [63,95],  [67,137], [26,15.62], [38,32.5],
    [32,20.88],[43.75,39.5],[52.25,53.4],[75,150],[77,168],
  [22.5,10.44],[40.8,32.5],[28.5,17.9],[24.5,12],[34.5,25.5],
  [26.5,14.06],[29,16.8],[31,20],[73,189],[60,155],
  [70,145],[72,149],[54,159],[64,133],[70,259],
  [65,121],[48,109],[18,6.23],[61.5,150],[19.5,5.94],
  [62.2,152],[20.5,8.7],[62.8,125],[20,8.06],[66,130],
  [19,6.5],[59,100],[61,126],[64,185],[70,200],
  [74,245],[66,170],[33,26],[72,250],[66,230],[71,200],
  [66,155],[19.5,8],[21,8.5],[72,145],[19,6.5],
  [29.75,21.5],[35,27.5],[38.5,31],[42,36],[45,43],
  [66,120],[19.25,5.1],[30.25,22.5],[35,26],[38.5,31],
  [42,36],[45,42],[63.5,100],[69,135],[69,190],
  [66,210],[60,66],[62,150],[61,112],[64,164],
  [69,190],[72,223],[30,37],[27,30],[46,59],
  [66,208],[44.5,35],[21,7.7],[27,15.8],[71.5,181],
  [60,78],[36,32],[52,55],[54,55],[20,9.3],
  [20,8.5],[20,7.5],[20.5,9.3],[21,9.6],[18,5.8],
  [20.5,9],[19,6.2],[20.5,7.1],[20.5,7.2],[19,6],
  [19,6.7],[67,148],[72,205],[69,140],[71,215],
  [67,155],[72,198],[75,225],[62,180],[64,145],
  [21,7.9],[19,6.4],[20.5,8.7],[73,260],[64,120],
  [65,140],[18.5,5.9],[19,6.3],[22,9.6],[25,11.1],
  [26.5,14.6],[28.8,16.2],[30,18.3],[34.8,24],[36.25,28],
  [40.5,33],[42,36.5],[45.5,39],[48.5,50],[52,59],
  [56,75],[61,98],[62,105.8],[20.75,7.8],[24,12.3],
  [25.5,14.5],[30,19.5],[34.5,26],[41,35],[43,39.8],
  [45.25,42],[47.75,50],[50.5,57],[51.8,58],[20.5,7.8],
  [23,12.9],[27.5,17.6],[31,23.5],[34,28],[36.5,32],
  [40,35.5],[42.25,39],[44,42.5],[46.5,48],[49.5,56],
  [51,60],[52,66],[54,73],[56,79],[59,95],
  [28,20]]:
> A1:=convert(S,matrix):
```

1) Find a least squares line fit to the ln-ln data which you obtain from A1.

We follow the same steps as in the powerlaws.mws file, copying and modifying the commands as necessary.

```
> A2:=map(ln, A1):
     #compute the log-log data
> A3:=map(evalf,A2):
     #decimal values
> rowdim(A3); #how many data points = number of rows in A3
```

157

```
> col2:=vector(rowdim(A3),1): #a column of 1's
  A4:=delcols(A3,2..2): #delete second col, means keep 1st!,
    #which is the ln(ht) data
  A:=augment(A4,col2): #this is the A matrix for least squares
> b:=delcols(A3,1..1):  #this is the ln(wt) data
> ATA:=evalm(transpose(A)&*A);
  ATb:=evalm(transpose(A)&*b);
  linsolve(ATA,ATb); #solve the normal equation for least
  #squares
```

$$ATA := \begin{bmatrix} 2189.571511 & 581.6672158 \\ 581.6672158 & 157 \end{bmatrix}$$

$$ATb := \begin{bmatrix} 2257.661748 \\ 586.3128449 \end{bmatrix}$$

$$\begin{bmatrix} 2.472169277 \\ -5.424630417 \end{bmatrix}$$

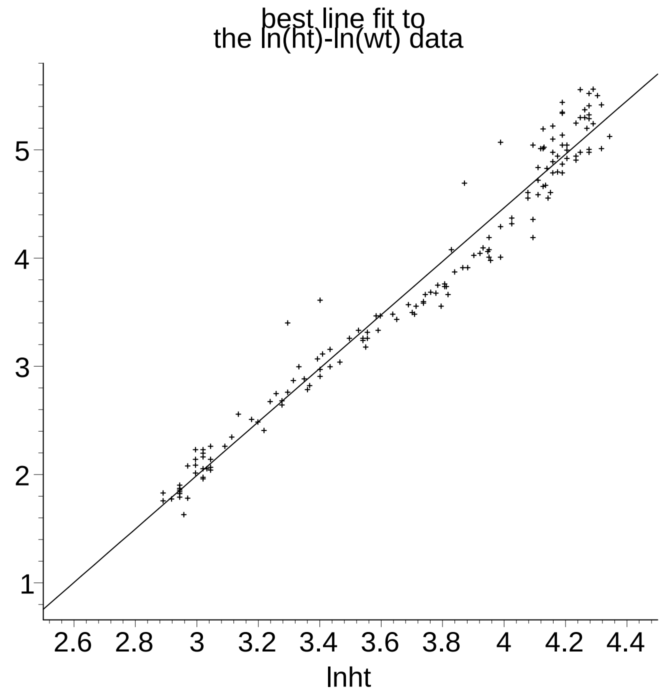
```
> evalm(inverse(ATA)&*ATb); #alternate method
```

$$\begin{bmatrix} 2.47216927 \\ -5.4246304 \end{bmatrix}$$

Thus the least squares line has slope 2.47216927 and vertical intercept -5.4246304.

2) Create a plot display which shows the ln-ln data and your least squares line fit.

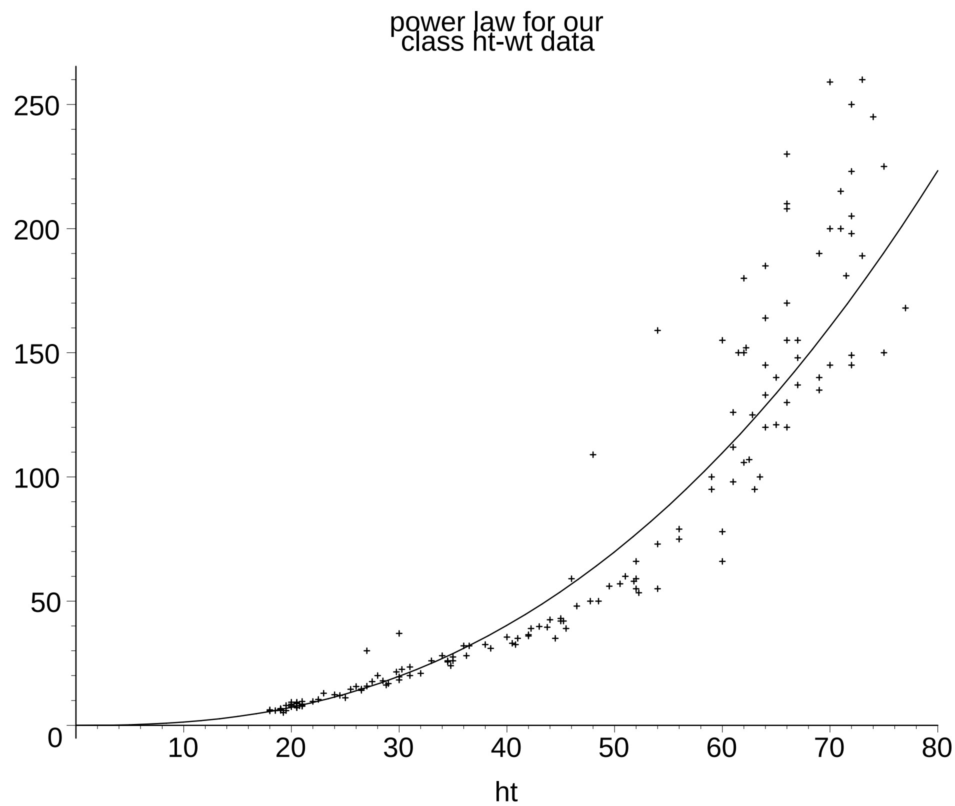
```
> lnlnplot:=pointplot({seq([A3[i,1],A3[i,2]],i=1..rowdim(A3))}):
    #the ln-ln data point plot
  line:=plot(2.47216927*lnht -5.4246304, lnht=2.5..4.5,color=black):
    #t = ln(h); I experimented on the appropriate t-interval
  display({lnlnplot,line},title=`best line fit to
  the ln(ht)-ln(wt) data`);
```



3) Deduce the optimal power law for our class height-weight data. Create a plot display which shows the graph of the power function and the pointplot of our original data.

The power is the slope of the line above, the proportionality constant is the exp of the line's vertical intercept:

```
> C:=exp(-5.4246304):
  p:=2.47216927:
  f:=ht->C*ht^p:  #our function
> realplot:=pointplot({seq([A1[i,1],A1[i,2]],i=1..rowdim(A3))}):powe
  rplot:=plot(C*ht^p,ht=0..80,color=black):
    #ht is height in inches
  display({realplot,powerplot},title=`power law for our
  class ht-wt data`);
```



4) What does your power law predict for the weights of "average" people, at heights of 5 feet, 5.5 feet, 6 feet, and 6.5 feet?

```
> f(60);  #average weight in pounds for 5 foot person,
    #according to our formula
  f(66);  #5.5 feet
  f(72);  #6 feet
  f(78);  #6.5 feet
```

$$109.6487865$$

$$138.7821316$$

$$172.0890641$$

$$209.7447425$$

```
>
```

5) On the internet, find the "normal" BMI range (for adults), i.e. the upper and lower "normal" BMI numbers, using the inch-pound system. Since in these units,

$$BMI = \frac{703\,w}{h^2}$$

the graph of heights and weights for which BMI has a constant value B is the parabola

$$w = \frac{1\,B\,h^2}{703}.$$

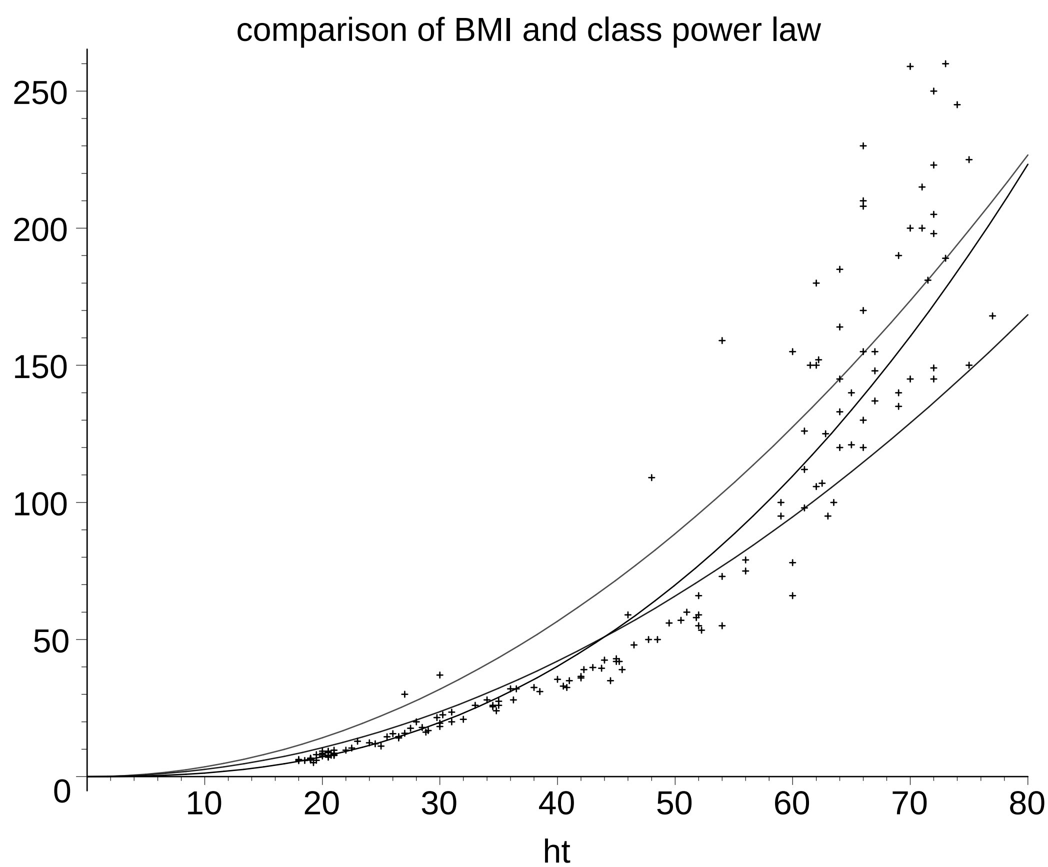
Create a single display containing our original data points, our power law curve, and the upper and lower parabolas for the "normal" BMI range. You can use different colors to identify the various curves. Our power law and the BMI curves should both fit the adult data pretty well. Explain why most children don't fit into the "normal" BMI range, based on our calculated power law and how it compares to a power law with p=2. How are the BMI standards adjusted for children? Is this consistent with our data?

According to the Center for Disease Control website of the U.S. government,

http://www.cdc.gov/nccdphp/dnpa/bmi/calc-bmi.html

normal range for BMI is between 18.5 and 24.9

```
> lowbmi:=ht->18.5*ht^2/703.0: #lower normal curve
  highbmi:=ht->24.9*ht^2/703.0: #upper normal curve
> lowplot:=plot(lowbmi(ht),ht=0..80, color=blue):
  highplot:=plot(highbmi(ht),ht=0..80, color=red):
    #the high and low parabolas defining high
    #and low endpoints of the "normal" BMI range
> display({realplot,powerplot,lowplot,highplot},title=
  'comparison of BMI and class power law');
```



Notice that our black power curve lies between the red and blue curves for adults, i.e. people over 5 feet tall. But for very tall people (over 78 inches), what our power law calls normal would be considered overweight by BMI, whereas for people under 50 inches, only three data points lie above the blue underweight BMI curve, so the many dozens of other people under 50 inches high would be classified as underweight, unless BMI ranges were changed for them. In practice, this is what happens - except the BMI ranges are given for ages, which act as a proxy for heights in children. If a power law using the power 2.5 was used, then BMI ranges would not depend so much on heights!

Note: It turns out a Belgian demographer, Adolphe Quetelet, also called the "Father of Statistics", originally proposed a power of p=2 for adults, based on his own data analysis during the early 1800's. In a footnote which history has forgotten, he said that a power of 2.5 is more appropriate if you want an approximation for people of all ages.