Math 2270-1
Fall 2005
Practice Exam

This exam is closed-book and closed-note. You may not use a calculator which is capable of doing linear algebra computations. In order to receive full or partial credit on any problem, you must show all of your work and **justify your conclusions.** There are 100 points possible, and the point values for each problem are indicated in the right-hand margin. Good Luck!

Well, actually, for this practice exam there are more than 100 points.

1a) Show that the following matrix equation has no solution

$$\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

(5 points)

1b) Find the least-squares solution to the problem in part (1a).

(10 points)

2) Let V be the span of the vectors

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

in R^3.

2a) Find an orthonormal basis for the plane V.

(10 points)

2b) Let $v = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$. Find the projection of v onto the subspace V, using your answer from (2 a)

(10 points)

2c) If you did your computations correctly, your answer to part (2b) should equal the matrix from number (1) multiplied by your least squares solution $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ to part (1b), i.e.

$$\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix}.$$

Explain why.

(5 points)

3a) Define a *linear transformation* L:V->W .

3b) Define the *kernel* and *image*, for a linear transformation L

3c) Prove that the kernel of a linear transformation is a subspace.

4) Let P2 be the space of polynomials in "t" of degree at most 2. Define T:P2->P2 by
$$T(f) = f'' + 4 f.$$

4a) Find the matrix for T, with respect to the standard basis $\{1, t, t^2\}$.

4b) Compute $T(c_0 + c_1 t + c_2 t^2)$ directly and then verify that the matrix you found in (4a) converts the coordinate vectors of inputs into coordinate vectors of outputs appropriately.

4c) Find bases for image(T) and kernel(T).

4d) verify the rank+nullity theorem for T

5) Let

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$$
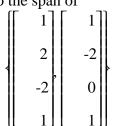
be a linear (matrix) map from R^2 to R^2. Let

$$B = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$$

be a non-standard basis for R^2. Find the matrix for L with respect to the basis B.

6) Find the least squares line fit for the following collection of 4 points: {[0,0],[1,2],[-1,-1],[-2,-3]}.

7) Find the orthogonal complement in R^4 to the span of

$$\left\{\begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix}\right\}$$

8) True-False: 4 points for each problem; two points for the answer and two points for the reason.

(32 points)

8a) Any collection of 3 polynomials of degree at most 2 must be linearly dependent.

8b) If the columns of a square matrix are orthonormal then so are the rows.

8c) The formula

$$[AB]^T = A^T B^T$$

holds for all square matrices

8d) If x and y are any two vectors in R^n, then $\|x+y\|^2 = \|x\|^2 + \|y\|^2$.

8e) If V is a 3-dimensional subspace of R^6, then the orthogonal complement to V is also 3-dimensional.

8f) If A is symmetric and B is symmetric, then so is A+B

8g) The functions $f(t)=t^2$ and $g(t)=t^3$ are orthogonal, with respect to the inner product

$$< f, g > = \int_{-1}^{1} f(t)\, g(t)\, dt$$

8h) If a collection of vectors is dependent, then any one of the vectors in the collection may be deleted without shrinking the span of the collection.