Name..
I.D. number...

Math 2270-1
Practice Exam
September 23, 2005

This is what it will say on the real exam:

This exam is closed-book and closed-note. You may not use a calculator which is capable of doing linear algebra computations. **In order to receive full or partial credit on any problem, you must show all of your work** and **justify your conclusions.** There are 100 points possible, and the point values for each problem are indicated in the right-hand margin. **Good Luck!**

This practice exam is longer than the real one will be, and has more than 100 points.

1) The following system of equations corresponds to the geometric configuration of three planes intersecting in a line:

$$2x + y + 3z = 2$$
$$x - y = 1$$
$$3x + 3y + 6z = 3$$

1a) Exhibit the augmented matrix corresponding to this system of three equations in three unknowns.
(5 points)

1b) Find the reduced row echelon form of your matrix from part (1a), and use it to solve the system.
(10 points)

1c) Verify that the direction vector for the line of intersection which you found in part 1b) is orthogonal (perpendicular) to each of the three plane normal vectors
(5 points)

2) Consider the matrix

$$B := \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & -1 \end{bmatrix}$$

2a) Use elementary row operations to find the inverse matrix to B. Check your answer by verifying that B times its inverse yields the identity matrix.
(15 points)

2b) Use your inverse matrix from 2b) or 2c) to solve the system:
(5 points)

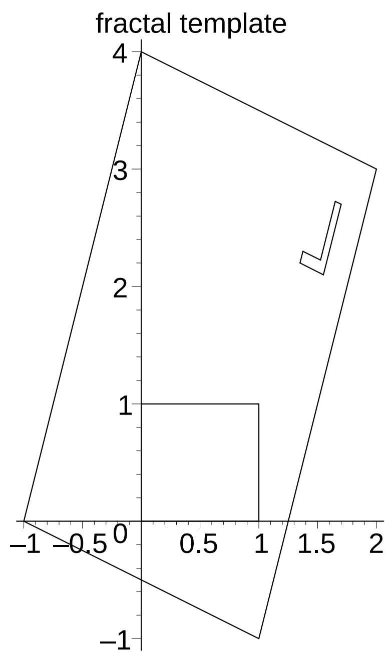
$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix}$$

3) Give the two definitions for a transformation (function) f(**x**) from R^n to R^m to be linear (one definition uses matrices, the other uses properties of the function). Explain why the two definitions are equivalent.

(15 points)

4) Write down the affine transformation which created the following "L-picture". (The parallelgram is the image of the unit square, by this affine transformation.)

(10 points)



5) Here is a matrix A:

$$A = \begin{bmatrix} 2 & -4 & -1 & 1 & -1 & 0 \\ 1 & -2 & -1 & 0 & -2 & -2 \\ 1 & -2 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 3 & 4 \end{bmatrix}$$

We consider the linear function, f(x)=Ax. Here is the reduced row echelon form of A:

$$\begin{bmatrix} 1 & -2 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5a) Find a basis for the image of f, which is a subset of the original six columns. Explain your reasoning.

(5 points)

5b) Express the sixth column of A as a linear combination of the basis vectors you found in part 5a.

(5 points)

5c) Find a basis for the kernel of f.

(10 points)

5e) State the theorem which relates the the dimensions of image(f), kernel(f) to the matrix dimensions. Verify that this theorem holds for the matrix A above. Explain why this theorem holds for every matrix.

(10 points)

6) True-False: 2 points for the right answer and 2 points for the justification, on each part:

(40 points total)

6a) If A and B are square matrices, then

$$(A-B)\,(A+B)=A^2-B^2$$

6b) The following identity is true, for invertible matrices A and B:

inverse(AB) = inverse(B)*inverse(A).

6c) If the matrix product AB = 0 (where 0 is the zero matrix), and if B is non-singular, then A must be the zero matrix.

6d) If A is a square matrix and A*A=A, then A=the identity matrix.

6e) If Ax=0 is a homogeneous system with an m by n matrix, and if the number of rows ‘‘m’’ is less than the number of columns ‘‘n’’, then there are always infinitely many solution vectors x.

6f) If 2u+3v+4w=5u+6v+7w then the subspace spanned by {u,v,w} is at most 2-dimensional.

6g) If AB=0 then BA=0 as well. (A and B are square matrices)

6h) Any three linearly independent vectors in R^3 are actually a basis for R^3.

6i) If the kernel of a matrix A consists of the zero vector only, then the column vectors of A must be linearly independent.

6j) If the vectors u,v,w are linearly dependent then w is a linear combination of u and v.