Math 2270-1
Wed Oct 5

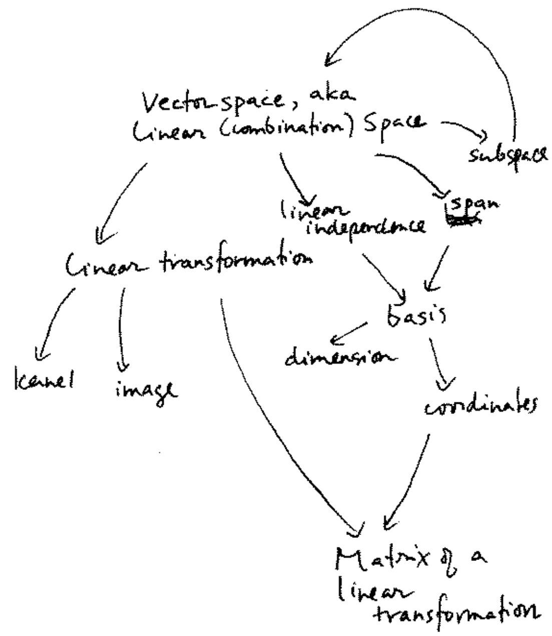
More HW for next Friday Oct 14!
<u>add</u> Chapter 4 review problems (True False), if true, prove; if false, find counterexample; all multiples of 4, i.e. 4, 8, ... 64

- pages 2, 3, 4 Tuesday.

Here's space to fill in details about why if

$$L : V \to W$$

is an isomorphism, then $L^{-1} : W \to V$ exists and is also an isomorphism. (These claims are made on the bottom of page 2 Tuesday, but there's no room for explanation.)



Matrix of a linear transformation

First note, compositions of linear transformations are linear:

Let $L : V \to W$ linear

$T : W \to Z$ linear

Then $T \circ L : V \to Z$

$(T \circ L)(v) := T(L(v))$ is linear

- check:

This diagram explains the matrix A of a linear transformation $L : V \to V$, with respect to a basis $\mathcal{B} = \{f_1, f_2, \dots f_n\}$ for V

$$\begin{tikzcd}
c_1f_1+c_2f_2+\dots+c_nf_n \quad V \arrow[r, "L"] \arrow[d, "{[v]_{\mathcal{B}}}"] & V \arrow[d, "{[\;]_{\mathcal{B}}}"] \\
\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} \quad \mathbb{R}^n \arrow[u, bend left] \arrow[r, "A"] & \mathbb{R}^n
\end{tikzcd}$$

$$A[\vec{v}]_{\mathcal{B}} = [L\vec{v}]_{\mathcal{B}}$$

in particular $\quad col_j(A) = A\vec{e}_j = A[f_j]_{\mathcal{B}} = [Lf_j]_{\mathcal{B}}$

" the j^{th} column of A is the $\mathcal{B}$ coord vector of Lf_j "

<u>example</u> (first of many).

page 3 Tuesday example continued.

$L : \mathbb{P}_2 \to \mathbb{P}_2$

$L(f) = f'$

Let $\mathcal{B} = \{1, x, x^2\}$.

- Find $A = [L]_{\mathcal{B}}$ → column by column / → directly.
- How are $\ker(L)$, $\operatorname{im}(L)$ related to A?