Math 2270-1
Fri 14 Oct

Maple Lab will be next Friday Oct 21

I need ht-wt data by Wed.

HW for Fri 10/21

5.1 1, 3, 5, 6, 7, 9, 12, 14, 15, 17, 18, 22, 23, 25, 26, 27, 29, 33

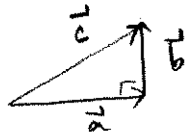
5.2 3, 4, 13, 17, 18, 27, 29, 32, 33, 35, 36, 42

5.3 1, 2, 7, 11, 15, 19, 2[illegible], 27, 31, 35, 40

§5.1 continued.

- finish Wed notes, pages 3-5, about <u>projection</u> onto subspaces

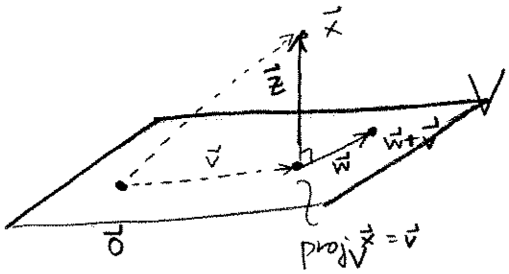
<u>Pythagorean Thm in $\mathbb{R}^n$</u>: Let $\vec{a}, \vec{b} \in \mathbb{R}^n$, $\vec{a}\cdot\vec{b} = 0$. Let $\vec{c} = \vec{a} + \vec{b}$



then $\|\vec{c}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2$

proof: $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = \vec{a}\cdot\vec{a} + 2\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b}$
$= \|\vec{a}\|^2 + \|\vec{b}\|^2$ $\blacksquare$

<u>Cor1</u> $\text{proj}_V \vec{x}$ is the nearest point in V to $\vec{x}$:



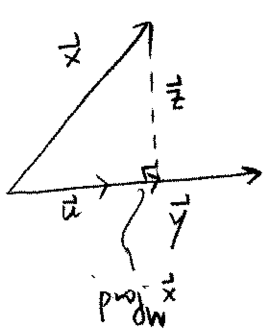
proof: Let $\vec{v} = \text{proj}_V \vec{x}$. Let $\vec{v}+\vec{w} \in V$. Let $\vec{z} = \vec{x} - \vec{v}$

$\|\vec{x} - (\vec{v}+\vec{w})\|^2 = \|(\vec{x}-\vec{v}) - \vec{w}\|^2$
$= \|\vec{z} - \vec{w}\|^2$
$= (\vec{z}-\vec{w})\cdot(\vec{z}-\vec{w})$
$= \|\vec{z}\|^2 + \|\vec{w}\|^2$!
$\geq \|\vec{z}\|^2$, equality iff $\vec{w} = \vec{0}$
$\vec{v}+\vec{w} = \vec{v}$.

<u>Cor2</u> Cauchy-Schwarz.

$|\vec{x}\cdot\vec{y}| \leq \|\vec{x}\|\|\vec{y}\|$ equality iff $\vec{x}$ and $\vec{y}$ are parallel (scalar mults)

<u>proof</u>



Let $W = \text{span}\{\vec{y}\}$
orthonormal basis $\{\vec{u}\}$, $\vec{u} = \frac{\vec{y}}{\|\vec{y}\|}$

$\vec{x} = \text{proj}_W \vec{x} + \vec{z}$
$\vec{x} = \vec{v} + \vec{z}$ $\quad \|\vec{x}\|^2 = \|\vec{v}\|^2 + \|\vec{z}\|^2$

so $\|\vec{x}\| \geq \|\vec{v}\| = \|(\vec{x}\cdot\vec{u})\vec{u}\| = |\vec{x}\cdot\vec{u}| = \left|\vec{x}\cdot\frac{\vec{y}}{\|\vec{y}\|}\right| = \frac{|\vec{x}\cdot\vec{y}|}{\|\vec{y}\|}$

so $\|\vec{x}\|\|\vec{y}\| \geq |\vec{x}\cdot\vec{y}|$. case of equality ✓

$\blacksquare$

Cor 3 the angle θ between $\vec{x}$ & $\vec{y}$ is defined to be

$$\theta = \arccos \frac{\vec{x}\cdot\vec{y}}{\|x\|\|y\|}$$, and this definition makes sense.

the application of this corollary to statistics is correlation coefficient r used to measure whether two data sets are correlated.

The example in the text is good. → go to page 196.

$\{x_1, x_2, \dots x_n\}$ $\quad \bar{x} := \frac{1}{n}\sum x_i$ av

$\{y_1, y_2, \dots y_n\}$ $\quad \bar{y} := \frac{1}{n}\sum y_i$ av

in book:

x_i : meat consumption in country i

y_i = cancer rate in country i

a statistics book would say

$$r = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2}\sqrt{\sum(y_i-\bar{y})^2}}, \quad -1 \le r \le 1. \qquad (=\cos\theta!)$$

if we normalize so that $\bar{x}$ & $\bar{y}=0$ [by subtracting $\bar{x}$ from each x_i & $\bar{y}$ from each y_i,

$$r = \frac{\vec{x}\cdot\vec{y}}{\|\vec{x}\|\|\vec{y}\|} = \cos\theta$$ [in $\mathbb{R}^n$, where n is # of data points].