

Math 2270-1  
Monday 10 Oct

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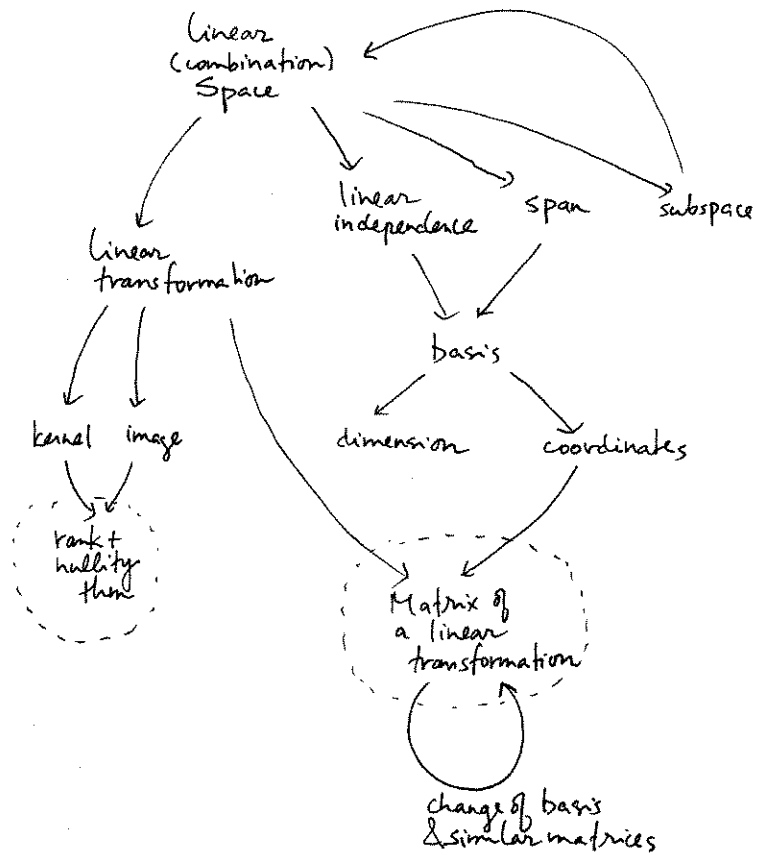
Assignment for next Maple project:

collect height-weight data from family & friends  
in 16 (especially babies & kids).  
we will use this data as part of next project.

Leftover pieces:

- rank + nullity theorem  
page 4 Tuesday 10/4
- Matrix of a linear transformation wrt basis  $\mathcal{B}$   
explanation page 2 Wed. 10/5  
example page 3 "

More examples in today's notes!



Example 3 page 174

on  $C^\infty(\mathbb{R}, \mathbb{R})$  consider

$$T(f) := f'' - 2f' - 3f$$

(note,  $T$  is linear)

Let  $V \subset C^\infty$ ,  $V = \text{span}\{\cos x, \sin x\}$

(note  $T: V \rightarrow V$ ).

(a) Find the matrix of  $T$  on  $V$ , wrt  $\mathcal{B} = \{\cos x, \sin x\}$

(b) Use it to find a solution to  $f'' - 2f' - 3f = \cos x$

$$A [f]_{\mathcal{B}} = [Tf]_{\mathcal{B}}$$

$$T(\cos x) = -\cos x + 2\sin x - 3\cos x$$

$$T(\sin x) = -\sin x - 2\cos x - 3\sin x$$

So

$$(a) \quad A = \begin{bmatrix} -4 & 2 \\ 2 & -4 \end{bmatrix}$$

$$(b) \quad A \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{20} \begin{bmatrix} -4 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{5} \\ -\frac{1}{10} \end{bmatrix}$$

So  $f(x) = -\frac{1}{5} \cos x - \frac{1}{10} \sin x$  will work!

could do directly:

$$\begin{aligned} -3 & \left( f = a \cos x + b \sin x \right) \\ -2 & \left( f' = -a \sin x + b \cos x \right) \\ 1 & \left( f'' = -a \cos x - b \sin x \right) \end{aligned}$$

$$L(f) = \cos x (-a - 2b + 3a) + \sin x (-b + 2a - 3b)$$

want =  $\cos x$

$$\begin{aligned} -4a - 2b &= 1 \\ 2a - 4b &= 0 \end{aligned}$$

same system

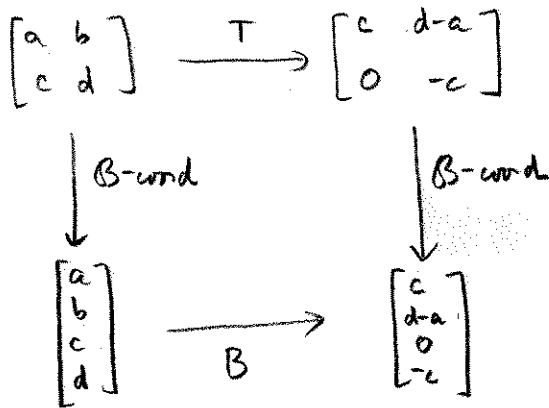
Example 7 p.170  $T: \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}$

$$T(M) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} M - M \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

- a) find matrix  ${}^B A$  of  $T$  wrt  $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$
- b) find image & kernel of  $B$
- c) find image & kernel of  $T$
- d) find rank & nullity of  $T$

$$T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & a \\ 0 & c \end{bmatrix} = \begin{bmatrix} c & d-a \\ 0 & -c \end{bmatrix}$$



d) rank  $T = 2$   
 nullity  $T = 2$   
 rank + nullity = 4.

so  $\ker(T) = \text{span} \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$   
 $\text{image}(T) = \text{span} \left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$

a)  $B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$

$\text{rref}(B) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$B\vec{x} = \vec{0}$ :

$x_4 = t$   
 $x_3 = 0$   
 $x_2 = s$   
 $x_1 = t$

$\vec{x} = t \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$\text{nullspace}(B) = \text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

b)  $\text{image}(B) = \text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

c)