Recall complex numbers, complex plane, geometric meaning of complex # addition, real scalar multiplication, complex # multiplication (moduli multiply, angles add).

encoded using Euler's formula, $e^{i\theta} = \cos \theta + i \sin \theta$

$z = re^{i\theta}$
$w = e^{i\phi}$

Then $zw = re^{i(\theta + \phi)}$

In particular, if $n$ is a counting number,

$z^n = (re^{i\theta})(re^{i\theta}) \cdots (re^{i\theta})$
$z^n = r^n e^{in\theta}$
$= r^n (\cos n\theta + i \sin n\theta)$

and $z^{-n} = \frac{1}{z^n} = \frac{1}{r^n} e^{-in\theta} = r^{-n} e^{-in\theta}$

so for $n \in \mathbb{Z}$ integers

\[ A \] $z^n = r^n e^{in\theta} = r^n (\cos n\theta + i \sin n\theta)$

DeMoivre's formula (all we really need for discrete dynamical systems)

What about roots?

\[ B \] $z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{\frac{i\theta}{n}}$ is a choice of a single $n^{th}$ root of $z$; there are $n-1$ others.

\textit{e.g.} find all cube roots of $z = -1$:

$-1 = 1 e^{i\pi}$

$\sqrt[3]{z} = \frac{1}{2} e^{i\frac{\pi}{3}}$ is one cube root of $-1$.

there are 2 more

\[ z^3 + 1 = 0 \quad \text{algebraically} \]

\textit{e.g.:} all solutions to $z^{10} = 1$
combining $A, B$, we see
that a sensible way to
define
\[ Z_{\text{min}} \]
is
\[ Z_{\text{min}} = (r_{\text{min}} e^{i\theta_{\text{min}}})^n \]
(if we use a different value of $Z_{\text{min}}$, we
will probably get a different
value of $Z_{\text{min}}$)

\[ Z = r e^{i\theta} \]

\[ Z^t = r e^{it\theta} = r^t (\cos \theta + i \sin \theta) \quad \text{if real} \]

interpolates De Moivre's formula, with an oo'ly dippable few

linear algebra with complex scalars (see C instead of IR)
Most concepts carry through:

* (Complex) vector space
* subspace
  * span
  * linear indep.
  * basis
  * row ops
  * matrix ops
  * det

...evils, vectors, eigenbasis, diagonalizability

that's what we've been doing already.

for example, as a complex vector space $C^1$ has dimension 1
as a real vector space it has dimension 2

How about $C^2$?

\[ \rightarrow \] Now return to Glucose-insulin model, and derive
a closed form expression for $[G(t)]$ which explains the spiral
(page 4-8 Wed 11/23 notes)