Math 2270-1
Monday Nov 28

Recall complex numbers, complex plane,
geometric meaning of
complex # addition
real scalar multiplication
complex # multiplication
(moduli multiply, angles add)

encoded using Euler's formula, $e^{i\theta} = \cos\theta + i\sin\theta$

$$z = re^{i\theta} = r(\cos\theta + i\sin\theta)$$
$$w = \rho e^{i\phi} = \rho(\cos\phi + i\sin\phi)$$

then $zw = r\rho e^{i(\theta+\phi)}$

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in particular, if n is a counting number

$$z^n = (re^{i\theta})(re^{i\theta})\cdots(re^{i\theta})$$
$$z^n = r^n e^{in\theta}$$
$$= r^n(\cos n\theta + i\sin n\theta)$$

and $z^{-n} = \dfrac{1}{z^n} = \dfrac{1}{r^n} e^{-in\theta} = r^{-n} e^{-in\theta}$

so for $n \in \mathbb{Z}$ ← integers

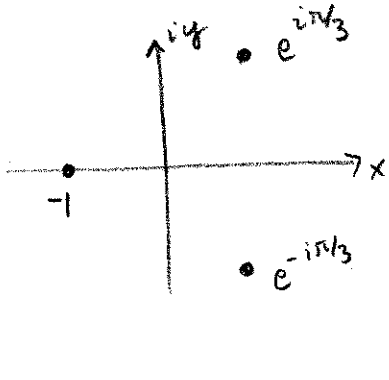
(A) $z^n = r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$ DeMoivre's formula [all we really need for discrete dynamical systems].

What about roots?

(B) $z^{1/m} = r^{1/m} e^{i\theta/m}$ is a choice of a single m^{th} root of z; there are $m-1$ others:

e.g. find all cube roots of $z = -1$:

$-1 = 1e^{i\pi}$ $\sqrt[3]{z} = 1^{1/3} e^{i\pi/3}$ is one cube root of -1.
there are 2 more



$z^3 + 1 = 0$
algebra...

e.g.: all sol'tns to $z^{10} = 1$

combining A, B, we see that a sensible way to define

$$z^{n/m}$$

is $z^{n/m} = \left(r^{1/m} e^{i\theta/m}\right)^n$

$z^{n/m} = r^{n/m} e^{i\frac{n}{m}\theta}$

(if we use a different value of $z^{1/m}$, we will probably get a different value of $z^{n/m}$)

$$\boxed{z^t := r^t e^{it\theta} = r^t(\cos t\theta + i\sin t\theta) \quad t \text{ real}}$$

interpolates De Moivre's formula, with an ∞'ly diffable fcn

linear algebra with complex scalars ($s \in \mathbb{C}$ instead of $\mathbb{R}$)
Most concepts carry through:

- (Complex) vector space
- subspace
- span
- linear indep.
- basis
- row ops
- matrix ops
- dets

⋮

evals, evects, eigenbasis, diagonalizability

that's what we've been doing already.

for example, as a complex vector space $\mathbb{C}$ has dimension 1
as a real vector space it has dimension 2
How about $\mathbb{C}^2$?

NOW return to Glucose-insulin model, and derive a closed from expression for $\begin{bmatrix} G(t) \\ H(t) \end{bmatrix}$ which explains the spiral

(page 4-5 Wed 11/23 notes)