Math 2270-1
Tuesday Nov. 22

Our next Maple lab will be postponed from the Monday after Thanksgiving

- Finish diagonalization thm from Monday (pages 2-3)
- Diagonalization for general linear transformations, example page 3 Monday

Computing A^t if A is similar to a diagonal matrix

More general fact:

if $B = S^{-1}AS$

then $B^2 = S^{-1}A\underbrace{SS^{-1}}_{I}AS = S^{-1}A^2S$

by induction,

$B^t = S^{-1}A^tS$

→ this is useful on 7.4 #54

So, if $\Lambda = S^{-1}AS$

↑ diagonal, $\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$.

then $S\Lambda S^{-1} = A$

$S\Lambda^t S^{-1} = A^t$

↑

$$\Lambda^2 = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}\begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix} = \begin{bmatrix} \lambda_1^2 & & & 0 \\ & \lambda_2^2 & & \\ & & \ddots & \\ 0 & & & \lambda_n^2 \end{bmatrix} \; ; \; \Lambda^t = \begin{bmatrix} \lambda_1^t & & & 0 \\ & \lambda_2^t & & \\ & & \ddots & \\ 0 & & & \lambda_n^t \end{bmatrix}$$

(by induction).

example: Find A^t $(t \in \mathbb{N})$

for $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

Complex eigenvalues/eigenvectors example
Math 2270-1
Tuesday November 22, 2005

Glucose-insulin model (see discussion on page 340 of the text.)

Let G(t) be the excess glucose concentration (mg of G per 100 ml of blood, say) in someone's blood, at time t hours. Excess means we are keeping track of the difference between current and equilibrium ("fasting") concentrations. Similarly, Let H(t) be the excess insulin concentration at time t. When blood levels of glucose rise, say as food is digested, the pancreas reacts by secreting insulin in order to utilize the glucose. Researchers have developed mathematical models for the glucose regulatory system. Here is a simplified (linearized) version of one such models. It would be meant to apply between meals, when no additional glucose is being added to the system:

```
> restart:with(linalg):with(plots):
```

$$G(t+1) = a\,G(t) - b\,H(t)$$
$$H(t+1) = c\,G(t) + d\,H(t)$$

Explain (understand) the signs of the matrix coefficients:

A particular choice of constants could lead to a model for you! For example

$$\begin{bmatrix} G(t+1) \\ H(t+1) \end{bmatrix} = \begin{bmatrix} 0.9 & -0.4 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} G(t) \\ H(t) \end{bmatrix}$$

Now let's solve the initial value problem, say right after a big meal, when

$$\begin{bmatrix} G(0) \\ H(0) \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

Of course, we know that for

```
> A:=matrix(2,2,[.9,-.4,.1,.9]);
```

$$A := \begin{bmatrix} 0.9 & -0.4 \\ 0.1 & 0.9 \end{bmatrix}$$

The solution at time t is just given by A^t times the initial vector. So we can make a table of values, but it won't help us understand the big picture:

```
> v:=vector([100,0]):
  for i from 1 to 30 do
  G[i]:=evalm(A^i&*v)[1]:
  H[i]:=evalm(A^i&*v)[2]:
  print(i,G[i],H[i]);
  od:
>
```

1, 90.0, 10.0

2, 77.00, 18.00

3, 62.100, 23.900

4, 46.3300, 27.7200

5, 30.60900, 29.58100

6, 15.715700, 29.683800

7, 2.2706100, 28.2869900

8, -9.27124700, 25.68535200

9, -18.61826310, 22.18969210

10, -25.63231363, 18.10889658

11, -30.31264090, 13.73477556

12, -32.77528704, 9.330033912

13, -33.22977189, 5.119501819

14, -31.95459544, 1.284574447

15, -29.27296567, -2.039342541

16, -25.52993208, -4.762704854

17, -21.07185693, -6.839427577

18, -16.22890021, -8.262670512

19, -11.30094198, -9.059293481

20, -6.547130390, -9.283458332

21, -2.179034020, -9.009825538

22, 1.642799596, -8.326746386

23, 4.809218190, -7.329791787

24, 7.260213086, -6.115890789

25, 8.980548094, -4.778280402

26, 9.993805445, -3.402397552

27, 10.35538392, -2.062777252

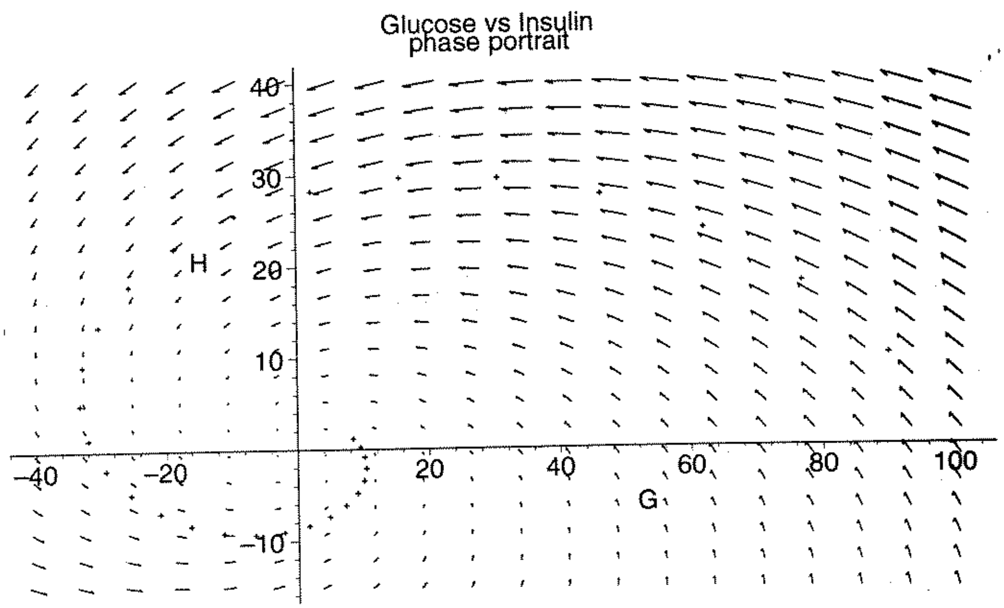
28, 10.14495643, -0.8209611340

29, 9.458845240, 0.2756306220

30, 8.402708470, 1.193952084

We get a better understanding if we plot the points onto the G-H plane, along with the **deviation vector** field:

```
> pict1:=fieldplot([-.1*G-.4*H,.1*G-.1*H],G=-40..100,H=-15..40):
  soltn:=pointplot({seq([G[i],H[i]],i=1..30)}):
  display({pict1,soltn},title=`Glucose vs Insulin
  phase portrait`);
```



Can we get a nice, closed form for the solution? Let's try our eigenvalue, eigenvector approach:

```
> with(linalg):
> eigenvects(A);
```

[0.9 − 0.2000000000 *I*, 1, {[-2.000000000 − 0. *I*, 0. − 1. *I*]}],

[0.9 + 0.2000000000 *I*, 1, {[-2.000000000 + 0. *I*, 0. + 1. *I*]}]

Can we use this to find a closed form solution to the problem, and to understand it?.....