Math 2270-1

Final Exam Review Information

December 7, 2005

The usual problem session time and location, 9:40-10:30 Thursday December 8 (tomorrow) in LCB 121, will primarily be for homework due this week, but you can also bring up other questions then. I have will my usual office hours in LCB 204 next week (MW 1-1:50, T 11-11:50), as well as an additional "special" time: Thursday, 9-11. Our exam is Friday morning, December 16th, 8-10 a.m., in our usual classroom JTB 120. I will let you stay until 10:30 a.m. Bring your winter wear since our classroom seems to be cold!!!

The exam will be comprehensive, except for section 8.3 on singular value decomposition. Precisely, you can expect anything from chapters 1-8.2, as well as the Kolman material on conic sections/quadric surfaces, and the affine transformation material related to fractals. In addition to being able to do computations, you should know key definitions, the statements of the main theorems, and why they are true. The exam will be a mixture of computational and theoretical questions. As on the midterms, there will be some true-false questions drawn from course material.

Exam material will be weighted towards topics which have not yet been tested, i.e. chapter 6-8 material.

I am handing out the actual final exam I gave the last time I taught 2270. A copy, with solutions, will be posted on our web page. Of course, our exam will be different!

Matrix algebra in R^n. (Chapters 1-2) (implicitly used in all other topics)

- Linear systems
- and matrix equations Ax=b
  - intersecting hyperplane interpretation (linear system interpretation)
  - linear combination of columns interpretation
- rref(A|b) and rref(A): how to compute, how to use.
- matrix transformations f(x)=Ax, and affine transformations f(x)=Ax+b.
  - geometric properties (i.e. parallel lines get mapped to parallel lines, translations and scalings of any set are transformed into translations and scalings of the transformed set.)
- geometric transformations (scalings, rotations, projections, reflections, shears)
- inverse transformations and inverse matrices
- composition of transformations and matrix products
- matrix algebra (i.e. commutative, associative, distributive properties with addition and multiplication of matrices)

Linear Spaces, (Chapters 3-4) :Chapter 3 was about R^n, and in Chapter 4 we generalized these ideas to general linear (vector) spaces.

Definitions:

Linear space
subspace
Linear transformation
domain
codomain
kernel
image
rank
nullity
linear isomorphism
linear combination
span
linear dependence, independence
basis
dimension
coordinates with respect to a basis
matrix of a linear transformation for a given basis

Theorems:

results about dimension: e.g. if dim(V)=n, then more than n vectors are ?, fewer than n vectors cannot ?, n linearly independent vectors automatically ?, n spanning vectors automatically are ?

also, if a collection of vectors is dependent, it may be culled without decreasing the span; if a vector is not in the span of a collection of independent vectors, it may be added to the collection without destroying independence.

the kernel and image of linear transformations are subspaces.

rank plus nullity equals ?

A linear transformation is an isomorphism if and only if ?

Isomorphisms preserve ?

Computations:

Check if a set is a subspace (also, subspaces of R^n.)
Check if a transformation is linear
Find kernel, image, rank, nullity of a linear transformation
Check if a set is a basis; check spanning and independence questions.
Find a basis for a subspace
Find coordinates with respect to a basis
Find the matrix of a linear transformation, with respect to a basis
Use the matrix of a linear transformation to understand kernel, image
Compute how the matrix of a linear trans changes if you change bases

Orthogonality (Chapter 5)

Definitions:

orthogonal
magnitude
unit vector
orthonormal collection
orthogonal complement to a subspace
orthogonal projection
angle
correlation coefficient (not on exam, but interesting)
orthogonal transformation, orthogonal matrix
transpose
least squares solutions to Ax=b
inner product spaces

Theorems

Pythagorean Theorem
Cauchy-Schwarz Inequality
Any basis can be replaced with an orthnormal basis (Gram Schmidt)
Algebra of the transpose operation
symmetric, antisymmetric
algebra of orthogonal matrices
Orthogonal complement of the orthogonal complement of V is V!

Computations

find coordinates when you have an orthonormal basis (in any inner product space)
Gram-Schmidt (in any inner product space)
A=QR decomposition
orthogonal projections (in any inner product space)
least squares solutions
application to best-line fit for data
find bases for the four fundamental subspaces of a matrix

Determinants (Chapter 6)

Definitions:

recursive definition of determinant
what is proof by induction?

Theorems:

determinant can be computed by expanding down any column or across any row
(You don't need to know the proof of this theorem!)
effects of elementary row operations (or column ops) on the determinant of a matrix
area/volume of parallelepipeds and determinants
adjoint formula of the inverse, and Cramer's rule
determinant of product is product of determinants
A is invertible if and only if its determinant is non-zero

Computations:

determinants by row ops or original definition
inverse matrices via adjoint formula; Cramer's rule for solving invertible systems.
computing areas or volumes of parallelepipeds.
the area or volume expansion factor of a linear transformation

Eigenvector concepts and applications (Chapters 7-8)

Definitions:

eigenvalue
eigenvector
characteristic polynomial
eigenspace
geometric and algebraic multiplicity of eigenvalues
eigenbasis for A
A is diagonalizable
all of the above definitions using complex scalars and vectors
Euler's formula for $\mathbf{e}^{(i\,\theta)}$.
discrete dynamical system with transition matrix A
regular transition matrix
quadratic form
conic section, quadric surface

Theorems:

Similar matrices have identical characteristic polynomials (so same eigenvalues), and their eigenspaces with equal eigenvalues have the same dimension.
A is diagonalizable iff the geometric and algebraic multiplicities of each eigenvalue agree.
if A is n by n and has n distinct eigenvalues, then A is diagonalizable. In all other cases, see above!
$\mathbf{e}^{(i\,\alpha)}\mathbf{e}^{(i\,\beta)} = \mathbf{e}^{(i\,(\alpha+\beta))}$.
When is the zero vector a (asymptotically) stable equilibrium for a discrete dynamical system?
if A is real and symmetric then A has an orthonormal eigenbasis (Spectral Theorem)
any quadratic form can be diagonalized - i.e. there is an orthogonal change of coordinates which eliminates all cross terms.

Computations:

find characteristic polynomial, eigenvalues, eigenspace bases.
above, when eigenvalues are complex.
can you tell if two given matrices are similar?
find a closed form for the solution x(t) to a discrete dynamical system whose transition matrix A can be diagonalized, depending on x(0).
identify and graph a conic or quadric surface by finding the equation its coordinates satisfy in a rotated basis.