Name	
I.D. number.	

Math 2270-1

Exam 2

November 4, 2005

This exam is closed-book and closed-note. You may not use a calculator which is capable of doing linear algebra computations. In order to receive full or partial credit on any problem, you must show all of your work and **justify your conclusions.** There are 100 points possible, and the point values for each problem are indicated in the right-hand margin. Good Luck!

1a) Consider the matrix system

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}.$$

Find the least squares solution to this problem.

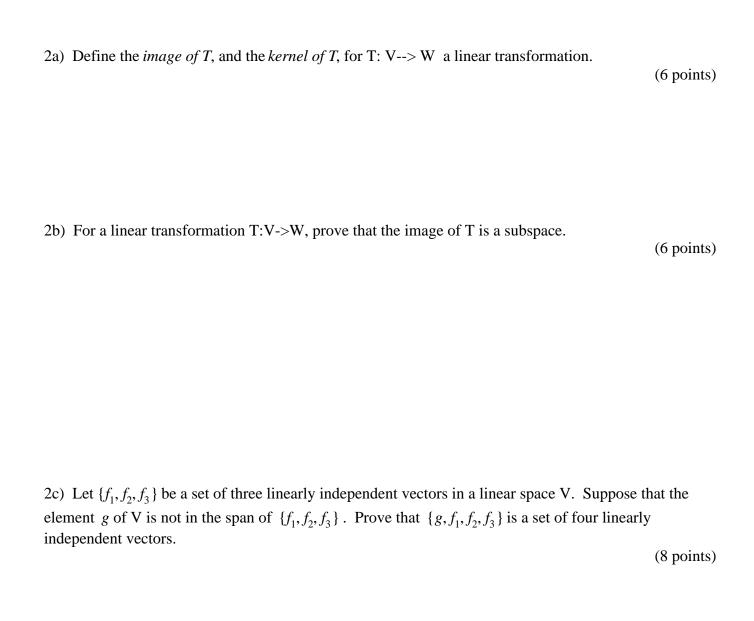
(10 points)

1b) The matrix system in part (1a) could have arisen as a least squares problem in the context of trying to get a best-line approximation to a data set consisting of the 3 points

$$\left\{ \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \end{bmatrix} \right\}.$$

Explain the meaning of "r" and "s" in this case. Then, sketch the three points below and the least-squares line fit.

(10 points)



3) Let V be the two-dimensional function vector space with basis $\beta = \{e^t, e^{(-t)}\}$. Let T be the linear transformation from V to V defined by

$$T(f) = f' + 3 f$$

("T of f is the derivative of f plus 3 times f").

3a) Find the matrix B for T with respect to the basis β. (Hint: it's diagonal!)

(6 points)

3b) Another basis for V is given by $\kappa = \{\cosh(t), \sinh(t)\}$. Recall that $\cosh(t) = \frac{1}{2} \mathbf{e}^t + \frac{1}{2} \mathbf{e}^{(-t)}$ and $\sinh(t) = \frac{1}{2} \mathbf{e}^t - \frac{1}{2} \mathbf{e}^{(-t)}$. What is the change of basis matrix S which converts κ -coordinates into β -coordinates?

(7 points)

3c) Find the matrix for the linear tranformation T(f), with respect to the basis κ .

(7 points)

4a) Find an orthonormal basis for R^3 by using the Gram-Schmidt algorithm on the three vectors

$$\left\{ \begin{bmatrix} -1\\0\\1 \end{bmatrix}, \begin{bmatrix} 0\\1\\0 \end{bmatrix}, \begin{bmatrix} 1\\1\\0 \end{bmatrix} \right\}$$

Hint: The first two vectors in the set are already orthogonal to each other.

(10 points)

4b) What is the A = QR factorization for the matrix

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(10 points)

5) True-False: 5 points for each problem; two points for the correct answer and three points for the explanation.

(20 points)

5a) If $\{u,v,w,z\}$ is an orthonormal collection of vectors, then ||u+v+w+z||=2.

5b) If A is an invertible matrix, then A^{T} is too. In fact,

$$[A^T]^{-1} = [A^{-1}]^T.$$

5c) If T is the projection in \mathbb{R}^3 to the plane x+y+z=0, then there is a basis for \mathbb{R}^3 for which the matrix of this projection transformation is

$$\begin{bmatrix}
 1 & 0 & 0 \\
 0 & 1 & 0 \\
 0 & 0 & 0
 \end{bmatrix}$$

5d) If the dimension of V is 6 and the dimension of W is 2, and if L:V-->W is linear, then the dimension of the kernel of L is at most 4.