Math 2270-1
Monday 29 Aug. §1.2

On Friday Prof. Alfeld showed you a selection of (surprising) applications of the general linear system

$$LS \quad \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

a_{ij} coefficients, known
b_i right hand sides, known.
x_j unknowns.

Many of these, and also different, examples will appear throughout the course.
For example, this weeks' HW includes small examples of

- polynomial interpolation
- Leontief input-output models
- chemical equation balancing.
- physics momentum problem

Example: set up §1.1 #21.

Today: Gauss-Jordan elimination (did example Wed.)

greatest mathematician ever?

- algebra
- probability
- geometry

p. 19 story of how Gauss used "least squares" solution to 17 linear eqtns in ? unknowns, (and invented) to calculate orbit of Ceres ... (also p. 8, Chinese knew this algorithm for solving linear systems 2000 years earlier.)

Let's try solving (§1.2 #16)

$$3x_1 + 6x_2 + 9x_3 + 5x_4 + 25x_5 = 53$$
$$7x_1 + 14x_2 + 21x_3 + 9x_4 + 53x_5 = 105$$
$$-4x_1 - 8x_2 - 12x_3 + 5x_4 - 10x_5 = 11$$

by hand (?!) ~ to understand how to reduce matrices, since "reduced row echelon form", the result of performing G-J elimination, is not only practically useful, but also key to understanding many "why's".

$$\left[\begin{array}{ccccc|c} 3 & 6 & 9 & 5 & 25 & 53 \\ 7 & 14 & 21 & 9 & 53 & 105 \\ -4 & -8 & -12 & 5 & -10 & 11 \end{array}\right]$$

We may use the 3 elementary row operations

- mult row by non-zero const
- interchange 2 rows
- replace a row by its sum with a multiple of another row.

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 0 & 5 & 6 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

rref

Reduced row echelon form: p.16

a) If a row has any non-zero entries, then the first non-zero entry is a 1, called the leading 1 of this row

b) If a column contains a (row's) leading 1, all other column entries are zero

c) as you move down the rows of the matrix the leading 1's move to the right by at least one column

does each matrix have exactly one rref?

ans: yes; easier to show in chapter 3.

Solution to original system:

What if rref had been

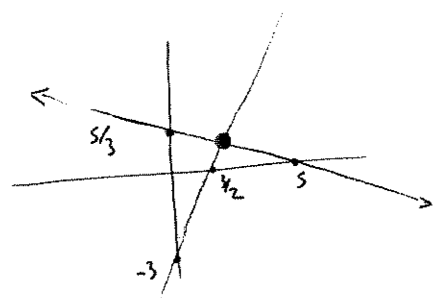
$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 0 & 5 & 6 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad ?$$

In HW (§1.2 #24,25) you ponder the fact that every elementary row operation can be reversed by an "inverse" row operation.

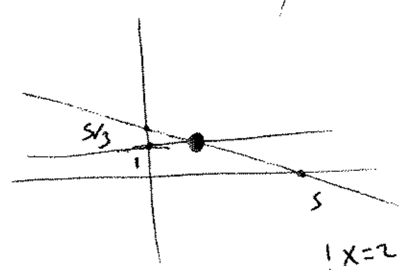
Since row ops correspond to equation operations,
and since a solution $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ to a linear system is also
a solution to a linear system obtained by doing an elementary eqtn operation,
<u>deduce</u>: elementary equation ops don't change soltn set.
So, the solution you read off from (correct) rref is the solution to original system.

<u>geometric example</u>

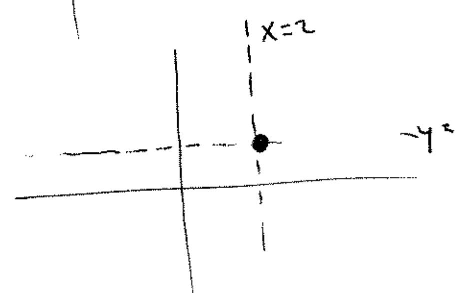
$$\begin{aligned} x + 3y &= 5 \\ 2x - y &= 3 \end{aligned} \qquad \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right]$$



$$\begin{aligned} x + 3y &= 5 \\ -7y &= -7 \end{aligned} \qquad \begin{array}{c} \\ -2R_1+R_2 \end{array} \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -7 & -7 \end{array}\right]$$



$$\begin{aligned} x &= 2 \\ y &= 1 \end{aligned} \qquad \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 1 & 1 \end{array}\right] \qquad \begin{array}{c} -3R_2+R_1 \\ \\ \end{array} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$



Math 2270-1
reduced row echelon form
Monday August 29, 2005

Let Maple compute rref for you, in big examples!

```
> with(linalg):  #load linear algebra library of procedures
> M:=matrix(4,6,[0,0,1,-1,-1,4,2,4,2,4,2,4,2,4,3,3,3,4,
          3,6,6,3,6,6]);
```

$$M := \begin{bmatrix} 0 & 0 & 1 & -1 & -1 & 4 \\ 2 & 4 & 2 & 4 & 2 & 4 \\ 2 & 4 & 3 & 3 & 3 & 4 \\ 3 & 6 & 6 & 3 & 6 & 6 \end{bmatrix}$$

```
> rref(M);
```

$$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1.2 #16)

```
> A:=matrix(3,6,[3,6,9,5,25,53,7,14,21,9,53,105,
     -4,-8,-12,5,-10,11]);
```

$$A := \begin{bmatrix} 3 & 6 & 9 & 5 & 25 & 53 \\ 7 & 14 & 21 & 9 & 53 & 105 \\ -4 & -8 & -12 & 5 & -10 & 11 \end{bmatrix}$$

```
> rref(A);
```

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 5 & 6 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1.2 #17)

```
> A:=matrix(5,6,[2,4,3,5,6,37,4,8,7,5,2,74,
        -2,-4,3,4,-5,20,1,2,2,-1,2,26,
        5,-10,4,6,4,24]);
```

$$A := \begin{bmatrix} 2 & 4 & 3 & 5 & 6 & 37 \\ 4 & 8 & 7 & 5 & 2 & 74 \\ -2 & -4 & 3 & 4 & -5 & 20 \\ 1 & 2 & 2 & -1 & 2 & 26 \\ 5 & -10 & 4 & 6 & 4 & 24 \end{bmatrix}$$

```
> rref(A);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \frac{-8221}{4340} \\ 0 & 1 & 0 & 0 & 0 & \frac{8591}{8680} \\ 0 & 0 & 1 & 0 & 0 & \frac{4695}{434} \\ 0 & 0 & 0 & 1 & 0 & \frac{-459}{434} \\ 0 & 0 & 0 & 0 & 1 & \frac{699}{434} \end{bmatrix}$$