Complex eigenvalues/eigenvectors exampl**e,**
REVISITED
Math 2270-2
Monday Nov 26, 2001

Glucose-insulin model (see discussion on page 339 of the text.)

Let G(t) be the excess glucose concentration (mg of G per 100 ml of blood, say) in someone’s blood, at time t hours. Excess means we are keeping track of the difference between current and equilibrium ("fasting") concentrations. Similarly, Let H(t) be the excess insulin concentration at time t. When blood levels of glucose rise, say as food is digested, the pancreas reacts by secreting insulin in order to utilize the glucose. Researchers have developed mathematical models for the glucose regulatory system. Here is a simplified (linearized) version of one such models. It would be meant to apply between meals, when no additional glucose is being added to the system:

```
> restart:with(linalg):with(plots):
```

$$\mathrm{G}(t+1)=a\,\mathrm{G}(t)-b\,\mathrm{H}(t)$$

$$\mathrm{H}(t+1)=c\,\mathrm{G}(t)+d\,\mathrm{H}(t)$$

A particular choice of constants could lead to a model for you! For example

$$\begin{bmatrix}\mathrm{G}(t+1)\\ \mathrm{H}(t+1)\end{bmatrix}=\begin{bmatrix}.9 & \text{-.4}\\ .1 & .9\end{bmatrix}\begin{bmatrix}\mathrm{G}(t)\\ \mathrm{H}(t)\end{bmatrix}$$

On November 16 we solved the initial value problem, say right after a big meal, when

$$\begin{bmatrix}\mathrm{G}(0)\\ \mathrm{H}(0)\end{bmatrix}=\begin{bmatrix}100\\ 0\end{bmatrix}$$

Of course, we know that for

```
> A:=matrix(2,2,[.9,-.4,.1,.9]);
```

$$A:=\begin{bmatrix}.9 & \text{-.4}\\ .1 & .9\end{bmatrix}$$

The solution at time t is just given by A^t times the initial vector. We made a table of values, by uncommenting the appropriate line in the do-loop below:

```
> v:=vector([100,0]):
  for i from 1 to 30 do
  G[i]:=evalm(A^i&*v)[1]:
  H[i]:=evalm(A^i&*v)[2]:
  #print(i,G[i],H[i]);
  od:
>
```

We used complex arithmetic (and evals and evects), as well as Euler’s formula to solve the IVP. Here was our data:

```
> eigenvects(A);
```

$$[.9+.2000000000\,I,\ 1,\ \{[\text{-}2.000000000+0.\,I,\ 0.+1.\,I]\}],$$
$$[.9-.2000000000\,I,\ 1,\ \{[\text{-}2.000000000-0.\,I,\ 0.-1.\,I]\}]$$

```
>
```

From which we deduced a closed form solution for G and H

```
> r:=sqrt(.85);
  theta:=arctan(.2/.9);
  Gt:=t->100*r^t*cos(theta*t);
  Ht:=t->50*r^t*sin(theta*t);
```

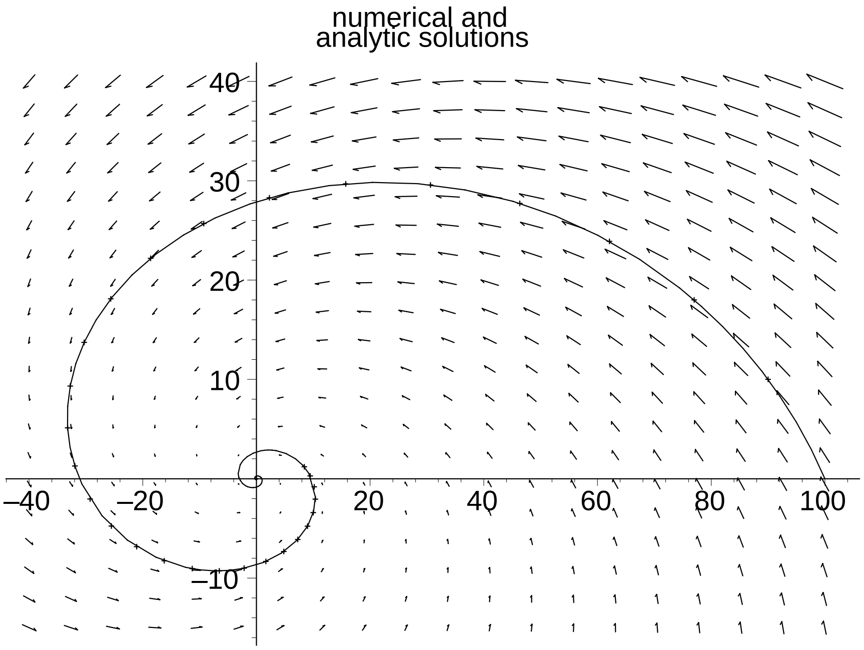
$$r := .9219544457$$

$$\theta := .2186689459$$

$$Gt := t \rightarrow 100\, r^{t} \cos(\theta\, t)$$

$$Ht := t \rightarrow 50\, r^{t} \sin(\theta\, t)$$

```
> soltn:=plot([Gt(t),Ht(t),t=0..100],color=black):
  pict1:=fieldplot([-.1*G-.4*H,.1*G-.1*H],G=-40..100,H=-15..40):
  actual:=pointplot({seq([G[i],H[i]],i=1..30)}):
  display({actual,pict1,soltn},title=‘numerical and
  analytic solutions‘);
```



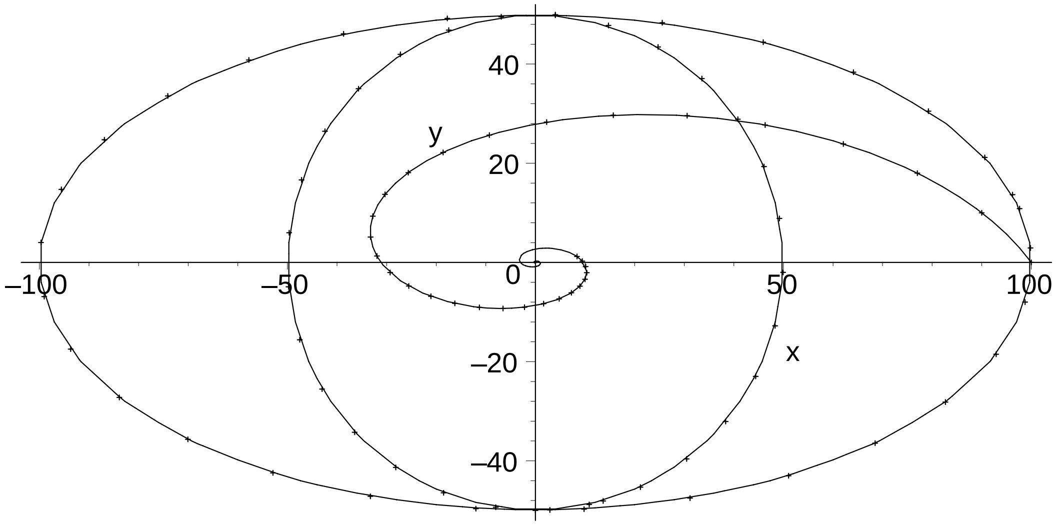
NEW TODAY:

We fit this special example into the general framework of page 3-4 of our class notes for Nov 26:

```
> p:=.9:
  q:=.2:
  phi:=arctan(2.0/9):
```

```
> circ1:=pointplot({seq([50*sin(k*phi),-50*cos(k*phi)],
  k=0..30)}):
  circ2:=implicitplot(x^2+y^2=2500,x=-100..100,
  y=-100..100,color=black):
  ellip1:=pointplot({seq([100*cos(k*phi),50*sin(k*phi)],
  k=0..30)}):
```

```
ellip2:=implicitplot(x^2/100^2+y^2/50^2=1,x=-100..100,
y=-100..100,color=black):
display({circ1,circ2,ellip1,ellip2,actual,soltn},title=‘general
decomposition picture‘);
```



```
>
```