MATHEMATICS 2270-1
Linear Algebra
Fall semester 2000

text: *Introductory Linear Algebra with Applications*, sixth edition, by Bernard Kolman
when: MTuWF 9:40-10:30
where: JTB 120

instructor: Prof. Nick Korevaar
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office hours: M 2-2:50 p.m., T 12:30-2:30 p.m., W 2-2:50 p.m., F 8:20-9:00 a.m.

course web page: www.math.utah.edu/~korevaar/2270fall00.html

prerequisites: Math 1210-1220, or Math 1250-1260, first year Calculus. Previous exposure to vectors, either in a multivariable Calculus course (e.g. 2210 or 1260) or in a Physics course, is helpful but not essential.

course outline: This is the first semester in a year-long sequence devoted to linear mathematics. Our topic this semester is linear algebra, a fundamental area of mathematics which is used to describe and study a multitude of subjects in science and life. The origins of this field go back to the algebra which one must solve to find the intersection of two lines in a plane, or of several planes in space, or more generally the solution set of one or more simultaneous "linear" equations involving several variables.

The course begins by studying linear systems of equations and the Gauss-Jordan method of systematically solving them. We see how to write these problems more succinctly in matrix form, we study the algebra of matrix operations, about inverses of non-singular square matrices, about determinants and their usefulness in solving linear problems. These topics comprise chapters 1-2 of the text.

In chapter 3 we review the linear geometry of $\mathbb{R}^2$, $\mathbb{R}^3$, and $\mathbb{R}^n$ and discuss the geometric meaning of matrices and determinants, as well as the dot and cross products and their geometric meanings. We recall complex-number algebra in Appendix A, and relate the algebra of complex multiplication and conjugation to their geometric interpretations in terms of rotations, scalings, and reflections in the complex plane.

At this point the course takes a turn towards the abstract as we study general vector spaces in chapters 4 and 6. Basically a vector space is a collection of objects (called vectors) which you can add and scalar multiply, such that certain arithmetic properties hold. From these arithmetic properties one develops notions such as linear independence, bases, dimension, subspaces, coordinates with respect to a basis, change of basis, linear transformations between vector spaces, kernel and range subspaces of linear transformations. We usually visualize vectors in the concrete example of $\mathbb{R}^n$, but in fact there are very natural spaces of functions and of solutions to certain (homogeneous linear) differential equations which are also vector spaces, so that these abstract concepts also apply to them. It is precisely because vector spaces appear in these different disguises that it is worthwhile to discuss them in this abstract way: as characterized by properties rather than by explicit descriptions. You will appreciate this more when you take Math 2280 and apply vector space theory to your study of linear differential equations.

We will discuss the notion of eigenvectors and eigenvalues for linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^n$, in chapter 5. These will also be used heavily in Math 2280.

The dot product in $\mathbb{R}^n$ lets one talk about orthogonality and orthogonal projections, and we discuss several applications related to this circle of ideas: Gram-Schmidt orthogonalization, methods of least squares (8.4), diagonalizing quadratic forms (8.8), rotating space to express conic sections and quadratic surfaces optimally (8.9-8.10). We will show there is also a natural "dot" product on certain function spaces, and use this to motivate Fourier series (appendix B.1) which you will return to in Math 2280.

Although we will use mostly real (scalar) vector spaces, it is often important in applications to allow complex numbers as your scalars, and we discuss this in Appendix A. If there is time at the end of the semester we will also spend a several lectures sketching some of the applications of our work to differential equations (8.6), as a way of forshadowing your work in Math 2280.

computer projects: There will be approximately 5 computer projects during the semester, to enhance and expand upon the material in the text. They will be written in the software package MAPLE. On MAPLE days we will meet in the Math Department Computer Lab located in the South Physics building, room 205. This building lies just north of the Math Department building JWB. We do not assume you have had any previous experience with this software and we will make the necessary introductions during the first visit to the lab.

tutoring center: The Math Department Tutoring Center is located in Mines 210, and is open for free tutoring from 8 a.m. to 8 p.m. on M-Th, and from 8 a.m to 2 p.m. on Friday. Some, but not all of the math tutors welcome questions from Math 2270 students. To see the times and specialities of various tutors, consult the web address www.math.utah.edu/ugrad/tutoring.html .

grading: There will be two midterms, a comprehensive final examination, and homework. (Homework assignments and other course information will be posted on the course web page.) Each midterm will count for 20% of your grade, homework Iincluding book and Maple assignments) will count for 30%, and the final exam will make up the remaining 30%. The book homework will be assigned daily and collected weekly, usually on Fridays. Maple projects will generally be due one week after they are assigned. A homework grader will partially grade your assignments. The value of carefully working homework problems is that mathematics (like anything) must be practiced and experienced to be learned.

It is the Math Department policy, and mine as well, to grant any withdrawl request until the University deadline of October 20.

ADA Statement: The American with Disabilities Act requires that reasonable accomodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations for the course.

Tentative Daily Schedule

exam dates fixed, daily subject matter approximated

W	23 Aug	1.1	linear systems
F	25 Aug	1.2	matrices
M	28 Aug	1.3	matrix multiplication
T	29 Aug	1.4	matrix operations
W	30 Aug	1.5	solving linear systems
F	1 Sept	1.6	matrix inverses
M	4 Sept	none	Labor Day
T	5 Sept	2.1	determinant definition
W	6 Sept	maple	begin project I
F	8 Sept	2.2	cofactor expansions
M	11 Sept	2.2	and applications
T	12 Sept	3.1	vectors in the plane
W	13 Sept	3.2	n-vectors
F	15 Sept	3.3-3.4	linear transformations
M	18 Sept	3.3-3.4	linear transformations
T	19 Sept	3.5	cross product
W	20 Sept	maple	begin project II
F	22 Sept	3.6	lines and planes
M	25 Sept	A.1	complex numbers
T	26 Sept	A.2	complex matrices
W	27 Sept	review	for exam 1
F	29 Sept	exam 1	1-3, Appendix A
M	2 Oct	4.1	real vector spaces
T	3 Oct	4.2	subspaces
W	4 Oct	4.3	linear independence
F	6 Oct	none	fall break day
M	9 Oct	4.4	basis and dimension
T	10 Oct	4.4	basis and dimension
W	11 Oct	4.5	homogeneous systems
F	13 Oct	4.6	matrix rank
M	16 Oct	4.7	coordinates and change of basis
T	17 Oct	4.7	continued
W	18 Oct	maple	begin project III
F	20 Oct	4.8	orthonormal bases

M	23 Oct	4.8-4.9	and orthogonal complements
T	24 Oct	4.9	orthogonal complements
W	25 Oct	8.4	least squares
F	27 Oct	8.4 & B.1	and inner product spaces
M	30 Oct	B.1	Fourier Series
T	31 Oct	6.1	linear transformations
W	1 Nov	maple	begin project IV
F	3 Nov	6.2	kernel and range
M	6 Nov	6.3	matrix of a linear transformation
T	7 Nov	6.3	continued
W	8 Nov	review	for exam
F	10 Nov	exam 2	4, 8.4, B.1, 6
M	13 Nov	5.1	eigenvalues and eigenvectors
T	14 Nov	5.1-5.2	diagonalization
W	15 Nov	5.2	diagonalization for symmetric matrices
F	17 Nov	extra	spectral theorem for symmetric matrices
M	20 Nov	extra	multivariable second derivative test
T	21 Nov	8.8	quadratic forms
W	22 Nov	none	semester break day
F	24 Nov	none	Thanksgiving break
M	27 Nov	8.9	conic sections
T	28 Nov	8.9-8.10	and quadric surfaces
W	29 Nov	maple	begin project V
F	1 Dec	8.10	quadric surfaces
M	4 Dec	8.6	differential equations
T	5 Dec	8.6	differential equations
W	6 Dec	8.6	differential equations
F	8 Dec	review all	for final exam
T	12 Dec	FINAL EXAM	entire course 9:15-11:15