Math 2250-004
Tues Jan 9

- Course introduction, continued with an example
- 1.2 Differential equations that can be solved by direct antidifferentiation

Announcements:
- Office Hours on T. usually 4:30-6:00 are 5:00-6:00 today in JWB 240
- Web page is up (Canvas later)
- Quiz tomorrow :)

Warm-up Exercise: Solve this initial value problem for position function $x(t)$

IVP $\begin{cases} x'(t) = 4\sin(2t) \\ x(0) = 0 \end{cases}$

e.g. $t$ = time sec
$x$ = position in meters

hint: $x(t) = \int 4\sin(2t)\,dt = -2\cos 2t + C$

('til 10:50)

$\left( \frac{d}{dt} -2\cos 2t = -2(-\sin(2t))\cdot 2 = 4\sin(2t) \checkmark \right)$

$x(0) = 0 = -2 + C$

Answer $x(t) = -2\cos 2t + 2$ ✓

(Question: What if DE was $x'(t) = 4\sin(2x)$ $x = x(t)$
cannot just antidiff w.r.t. $t$
because RHS depends on $x(t)$, which you don't know yet.)

• **important course goals:** understand some of the key differential equations which arise in modeling real-world dynamical systems from science, mathematics, engineering; how to find the solutions to these differential equations if possible; how to understand properties of the solution functions (sometimes even without formulas for the solutions) in order to effectively model or to test models for dynamical systems.

In fact, you've encountered differential equations in previous mathematics and/or physics classes:

• $1^{st}$ order differential equations: rate of change of function depends in some way on the function value, the variable value, and nothing else. For example, you've studied the population growth/decay differential equation for $P = P(t)$, and $k$ a constant, given by

$$P'(t) = k\,P(t)$$

and having applications in biology, physics, finance.

how fast P(t) changes

is proportional to P(t).

• $2^{nd}$ order DE's: Newton's second law (change in momentum equals net forces) often leads to second order differential equations for particle position functions $x = x(t)$ in physics.

$$m\,x''(t) = \text{net forces}$$

$$= \underbrace{f(t)}_{\text{e.g. external forcing function}} \underbrace{- k\,x(t)}_{\text{Hooke's Law}} - c\,x'(t) \qquad \leftarrow \text{damped mass spring}$$

drag coeff.

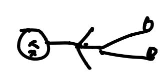
The following exercise is an illustration of how modeling and differential equations tie together...

Exercise 1) Newton's law of cooling is a model for how objects are heated or cooled by the temperature of an ambient medium surrounding them. In this model, the body temperature $T = T(t)$ changes at a rate $\}$ $T'(t)$

proportional to to the difference between it (T) and the ambient temperature $A(t)$. In the simplest models $A$ is constant.

a) Convert the description above into the differential equation for how the temperature of a heating or cooling object changes:

$$\frac{dT}{dt} = -k(T - A)\,.$$



T(t) temperature @ time t

ambient medium temperature A(t)

$$T'(t) = k_1 \cdot (T(t) - A(t))$$

how fast temp changes

If $T < A$ object is heating so $T'(t) > 0$. $T - A < 0$

$T'(t) = k_1(T - A)$ So $k_1 < 0$

b) Would the differential equation have been correct if we wrote $\dfrac{dT}{dt} = k_1(T - A)$ instead?

Yes. (In this case, the $k_1$ is neg.) so let $k = -k_1$, $k > 0$

c) Find all solution functions to the Newton's law of cooling differential equation

$$\frac{dT}{dt} = -k(T - A).$$

using separation of variables or the method of substitution. const.

$$\frac{T'(t)}{T(t)-A} = -k$$

$$\int \frac{T'(t)}{T(t)-A}\,dt = \int -k\,dt$$

$$= -kt + C_1$$

let $u(t) = T(t) - A$

$du = T'(t)\,dt$

$$\int \frac{du}{u} = \ln|u| + C_2$$

so $\ln|T(t)-A| + C_2 = -kt + C_1$

$\ln|T(t)-A| = -kt + C_3$

e e

$|T(t)-A| = e^{-kt}e^{C_3}$

$|T(t)-A| = C_4 e^{-kt}$

$T(t) - A = C_5 e^{-kt}$ $C_5 = \pm C_4$

$$\boxed{T(t) = A + Ce^{-kt}}$$

makes sense as $t \to \infty$.

d) Use the Newton's law of cooling model to partially solve a murder mystery: At 3:00 p.m. a deceased body is found. Its temperature is 70 ° F. An hour later the body temperature has decreased to 60 °. It's been a winter inversion in SLC, with constant ambient temperature 30 °. Assuming the Newton's law model, estimate the time of death.

$T = \cancel{A} + Ce^{-kt}$

$T = 30 + Ce^{-kt}$

$t = 0$ is when you found body

$T(1) = 60°$

$T(0) = 70°$

$A = 30$

get out of quiz free

bring soln

or get 5 more pts on quiz

**Section 1.2**: differential equations equivalent to ones of the form

$$y'(x) = f(x)$$

which we solve by direct antidifferentiation

$$y(x) = \int f(x)\, dx = F(x) + C.$$

Exercise 2 Solve the initial value problem

$$\frac{dy}{dx} = x\sqrt{x^2 + 4}$$
$$y(0) = 0$$

(like warmup)

HW rest of §1.1 : 27, 30, 33, 34.
§1.2 w/ 1.1 (antidiff)
2, 6, 7, 9, 16, 18
(maybe 24, 26, 33)