Syllabus for Math 2250-004 Differential Equations and Linear Algebra

Spring 2018

Instructor Professor Nick Korevaar
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office LCB 204, 801.581.7318
office hours M 2:00-3:00 p.m. LCB 204, T 4:30-6:00 location TBA, and by appointment. Also available after class (briefly).
Lecture MTWF 10:45-11:35 a.m. MWF in WEB L105, T in JWB 335, don't forget!

Laboratory sections with Dihan Dai, dai@math.utah.edu
2250-005 H 10:45-11:35 a.m. LCB 219
2250-006 H 9:40-10:30 a.m. AEB 310
with Jose Yanez, yanez@math.utah.edu
2250-015 H 10:45-11:35 a.m. JWB 308
2250-016 H 9:40-10:30 a.m. JWB 308

Course websites

Daily lecture notes and weekly homework assignments will be posted on our public home page.

http://www.math.utah.edu/∼korevaar/2250spring18

There are blank spaces in the notes where we will work out examples and fill in details together. Research has shown that class attendance with active participation - including problem solving and writing notes by hand - are effective ways to learn class material, for almost everyone. Passively watching a lecture is not usually effective. Class notes will be posted at least several days before we use them, so that you have ample time to print them out. Printing for math classes is free in the Math Department Rushing Student Center, in the basement of LCB. There will often be additional class discussion related to homework and lab problems.

Grades and exam material will be posted on our CANVAS course page; access via Campus Information Systems.

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Textbook *Linear Algebra and Differential Equations: with Introductory Partial Differential Equations*, by Edwards, Penney, and Haberman, a custom-printed textbook for the University of Utah: (ISBN: 13-9781269425575)

This text is a hybrid of the three texts: Differential Equations and Linear Algebra 3rd Edition, by Edwards and Penney; Applied Partial Differential Equations with Fourier Series and Boundary Value Problems, 5th edition, by Haberman; Elementary Linear Algebra, by Edwards and Penney. You should buy this version of the text if you plan to take the 4th semester in the new engineering math sequence, Math 3140, or the PDE course Math 3150, soon.

If your math courses will terminate with Math 2250, then the 3rd or 4th editions of the Differential Equations and Linear Algebra text by Edwards-Penney will suffice:

Differential Equations and Linear Algebra (4th Edition) (978-0134497181) brand new this year;

Differential Equations and Linear Algebra (3rd Edition) (978-0136054252) will go out of print at some point.

Final Exam logistics: ~~Thursday~~ Friday April 27, 10:30 a.m.-12:30 p.m., in our MWF classroom WEB L105. This is the University scheduled time and location.

Catalog description for Math 2250: This is a hybrid course which teaches the allied subjects of linear algebra and differential equations. These topics underpin the mathematics required for most students in the Colleges of Science, Engineering, Mines & Earth Science.

Prerequisites: Math 1210-1220 or 1310-1320 (or 1250-1260 or 1311-1321, i.e. single-variable calculus.) You are expected to have learned about vectors and parametric curves in one of these courses, or in Math 2210 or or Physics 2210 or 3210. Practically speaking, you are better prepared for this course if you've had elements of multivariable calculus in courses such as 1320, 1321, or 2210 and if your grades in the prerequisite courses were above the "C" level.

Grading - no worse than 90/80/70 scale

Math 2250-004 is graded on a curve. **note:** In order to receive a grade of at least "C" in the course you must earn a grade of at least "C" on the final exam. To see historical distributions of grades in my Math 2250 classes you can look at my old course home pages, on the exam pages.

Details about the content of each assignment type, and how much they count towards your final grade are as follows:

- Homework (10%): Homework from roughly three textbook sections is due every Wednesday at the beginning of class, based on lecture sections covered through the preceding Tuesday. All assignments will be posted on our public page, the Wednesday before they are due. Several problems per section will be randomly selected for grading. Two of a student's lowest homework scores will be dropped. Only hard-copy assignments will be accepted—no digital copies—and no late homework will be accepted.

- Quizzes (10%): At the end of most Wednesday classes, a short 1-2 problem quiz will be given, taking roughly 10 minutes to do. The quiz will cover relevant topics from the week's lectures, homework, and lab section work. Two of a student's lowest quiz scores will be dropped. There are no makeup quizzes. You will be allowed and encouraged to work together on these quizzes.

- Midterm exams (30%): Two class-length midterm exams will be given, On Friday February 16 and Friday March 30. Review for the exams will occur either in lecture or in the lab sections, in the days before the exams. No midterm scores are dropped.

- Final exam (30%): A two-hour comprehensive exam will be given at the end of the semester. As with the midterms, a practice final will be posted. Please check the final exam time, which is the official University scheduled time. It is your responsibility to make yourself available for that time, so make any arrangements (e.g., with your employer) as early as possible.

- Lab (20%): Every week a Teaching Assistant (TA) -directed lab section will be held. In lab, the TA will hand out problem worksheets and will facilitate student-led group work. The worksheet problems will provide guided practice with both basic methods, as well as longer in-depth problems with physical and engineering applications. Completed lab assignments will be due at the start of the following week's lab class. Credit will be given for both lab attendance and completed worksheets. Students should expect that worksheets will take additional time outside of lab to finish completely. None of the lab worksheet grades will be dropped. The TA's will be available for additional office hours.

Strategies for success

- Attend class and lab regularly.

- Read or skim the relevant text book sections and lecture note outlines *before* you attend class.

- Ask questions and become involved.

- Plan to do homework daily; try homework on the same day that the material is covered in lecture; do not wait until just before homework and lab reports are due to begin serious work.

- Form study groups with other students.

Students with disabilities

The University of Utah seeks to provide equal access to its programs, services and activities for people with disabilities. If you will need accommodations in the class, reasonable prior notice needs to be given to the Center for Disability Services, 162 Olpin Union Building, 581-5020 (V/TDD). CDS will work with you and the instructor to make arrangements for accommodations. All information in this course can be made available in alternative format with prior notification to the Center for Disability Services.

Learning Objectives for 2250

The goal of Math 2250 is to master the basic tools and problem solving techniques important in differential equations and linear algebra. These basic tools and problem solving skills are described below.

The essential topics

Be able to model dynamical systems that arise in science and engineering, by using general principles to derive the governing differential equations or systems of differential equations. These principles include linearization, compartmental analysis, Newton's laws, conservation of energy and Kirchoff's law.

Learn solution techniques for first order separable and linear differential equations. Solve initial value problems in these cases, with applications to problems in science and engineering. Understand how to approximate solutions even when exact formulas do not exist. Visualize solution graphs and numerical approximations to initial value problems via slope fields. Understand phase diagram analysis for autonomous first order differential equations.

Become fluent in matrix algebra techniques, in order to be able to compute the solution space to linear systems and understand its structure; by hand for small problems and with technology for large problems.

Be able to use the basic concepts of linear algebra such as linear combinations, span, independence, basis and dimension, to understand the solution space to linear equations, linear differential equations, and linear systems of differential equations.

Understand the natural initial value problems for first order systems of differential equations, and how they encompass the natural initial value problems for higher order differential equations and general systems of differential equations.

Learn how to solve constant coefficient linear differential equations via superposition, particular solutions, and homogeneous solutions found via characteristic equation analysis. Apply these techniques to understand the solutions to the basic unforced and forced mechanical and electrical oscillation problems.

Learn how to use Laplace transform techniques to solve linear differential equations, with an emphasis on the initial value problems of mechanical systems, electrical circuits, and related problems.

Be able to find eigenvalues and eigenvectors for square matrices. Apply these matrix algebra concepts to find the general solution space to first and second order constant coefficient homogeneous linear systems of differential equations, especially those arising from compartmental analysis and mechanical systems.

Understand and be able to use linearization as a technique to understand the behavior of nonlinear dynamical systems near equilibrium solutions. Apply these techniques to non-linear mechanical oscillation problems. (Additional material, subject to time availability: Apply linearization to autonomous systems of two first order differential equations, including interacting populations. Relate the phase portraits of non-linear systems near equilibria to the linearized data, in particular to understand stability.)

Develop your ability to communicate modeling and mathematical explanations and solutions, using technology and software such as Maple, Matlab or internet-based tools as appropriate.

Problem solving fluency

Students will be able to read and understand problem descriptions, then be able to formulate equations modeling the problem usually by applying geometric or physical principles. Solving a problem often requires specific solution methods listed above. Students will be able to select the appropriate operations, execute them accurately, and interpret the results using numerical and graphical computational aids.

Students will also gain experience with problem solving in groups. Students should be able to effectively transform problem objectives into appropriate problem solving methods through collaborative discussion. Students will also learn how to articulate questions effectively with both the instructor and TA, and be able to effectively convey how problem solutions meet the problem objectives.

Week-by-Week Topics Plan

Topic schedule is subject to slight modifications as the course progresses, but exam dates are fixed.

Week 1: 1.1-1.4; differential equations, mathematical models, integral as general and particular solutions, slope fields, separable differential equations.

Week 2: 1.4-1.5, EP 3.7, 2.1-2.2; separable equations cont., linear differential equations, circuits, mixture models, population models,equilibrium solutions and stability.

Week 3: 2.2-2.4; equilibrium solutions and stability cont., acceleration-velocity models, numerical solutions.

Week 4: 2.5-2.6, 3.1; numerical solutions cont., linear systems.

Week 5: 3.1-3.4; linear systems, matrices, Gaussian elimination, reduced row echelon form, matrix operations.

Week 6: 3.5-3.6; matrix inverses, determinants, review; **Midterm exam 1 on Friday February 16** covering material from weeks 1-6.

Week 7: 4.1-4.3; vector spaces, linear combinations in $\mathbb{R}^n$, span and independence, subspaces, bases and dimension.

Week 8: 4.4, 5.1-5.3; second-order linear DEs, general solutions, superposition, homogeneity and constant coefficients.

Week 9: 5.3-5.5; mechanical vibrations, pendulum model, particular solutions to non-homogeneous problems.

Week 10: 5.5-5.6, EP 3.7; forced oscillations and associated physical phenomena.practical resonance Laplace transforms, solving IVPs with transforms, partial fractions and translations.

Week 11: 10.1-3; Laplace transforms, solving IVPs with transforms, partial fractions and translations. **Midterm exam 2 on Friday March 30** covering material from weeks 7-11.

Week 12: 10.4-10.5, EP 7.6, 6.1-6.2 Unit steps, convolutions, impulse function forcing; eigenvalues, eigenvectors and diagonalizability.

Week 13: 6.1-6.2 continued; 7.1-7.3; first order systems of differential equations; framework for differential equations in which every DE is equivalent to a first order system of DE's. Matrix systems of DEs

Week 14: 7.3-7.4; solution algorithms and applications for first and second order systems of differential equations; input-output modeling and mechanical systems.

Week 15: 7.3-7.4 continued, and review. **Final exam Friday April 27, 10:30 a.m. - 12:30 p.m. in classroom WEB L105. This is the University scheduled time.**

• Pick up syllabus & copy of 1st week outline
• I expect our public page to be up some time this evening
• " " " CANVAS " " " later this week.

Math 2250-004 Week 1 notes

We will not necessarily finish the material from a given day's notes on that day. Or on an amazing day we may get farther than I've predicted. We may also add or subtract some material as the week progresses, but these notes represent an outline of what we will cover. These week 1 notes are for sections 1.1-1.3, with hints of 1.4.

Monday January 8:

- Course Introduction ✓
- 1.1: differential equations and initial value problems

  also touching on
- 1.3: slope fields for first order differential equations

• Go over course information on syllabus and course homepage:

http://www.math.utah.edu/~korevaar/2250spring18

• Note that there is a quiz this Wednesday on the material we cover today and tomorrow, and that your first lab meeting is this Thursday. Your first homework assignment will be due next Wednesday, January 17.

Then, let's begin!

• What is an n^{th} order differential equation (DE)?

any equation involving a function $y = y(x)$ and its derivatives, for which the highest order *derivative appearing in the equation is the n^{th} one, $y^{(n)}(x)$; i.e. any equation which can be written as*

$$F\big(x, y(x), y'(x), y''(x), \ldots y^{(n)}(x)\big) = 0.$$

Exercise 1: Which of the following are differential equations? For each DE determine the order.

a) For $y = y(x)$, $(y''(x))^2 + \sin(y(x)) = 0$ yes! 2nd order

b) For $x = x(t)$, $x'(t) = 3\,x(t)\,(10 - x(t))$. yes! 1st order

means same

c) For $x = x(t)$, $x' = 3\,x\,(10 - x)$. yes!

d) For $z = z(r)$, $z'''(r) + 4\,z(r)$. no! not an equation (no "=" sign)

e) For $y = y(x)$, $y' = y^2$. yes! 1st order

short for $y'(x) = (y(x))^2$

f) $y' = y^{(2)}$ for $y(x)$ Yes! 2nd order!!

Definition: *A* <u>*solution*</u> *function* $y(x)$ *to a first order differential equation* $F(x, y, y') = 0$ *on the interval I is any function* $y(x)$ *which, when substituted into the differential equation, yields a true identity.*

Definition: *If the solution function also satisifies* $y(x_0) = y_0$ *for a specified* $x_0 \in I$ *and* $y_0 \in \mathbb{R}$*, then* $y(x)$ *is called a* <u>*solution to the initial value problem*</u> *(IVP)*

$$\text{IVP} \begin{cases} F(x, y, y') = 0 \\ y(x_0) = y_0 \end{cases}.$$

<u>Exercise 2</u>: Consider the differential equation $\frac{dy}{dx} = y^2$ from (1e).

DE $y'(x) = (y(x))^2$

<u>2a</u>) Show that functions $y(x) = \frac{1}{C - x}$ solve the DE (on any interval not containing the constant C).

plug $y(x)$ into the DE
does it yield a true identity.

$y'(x) = (y(x))^2$
LHS RHS

we got lucky!
$y(x) = x^2$
LHS $y'(x) = 2x$
RHS $y^2(x) = x^4$
LHS $\neq$ RHS not a soltn.

if $y(x) = \frac{1}{C-x}$

$\Rightarrow$ LHS $y'(x) = (C-x)^{-2} = \frac{1}{(C-x)^2}$

$\Rightarrow$ RHS $= y(x)^2 = \frac{1}{(C-x)^2}$

[$y(x) = (C-x)^{-1}$
$\Rightarrow y'(x) = -1 \cdot (C-x)^{-2}(-1)$]

Since LHS = RHS, $y(x)$ solves the DE.

<u>2b</u>) Find the appropriate value of *C* in the collection of solution functions $y(x) = \frac{1}{C - x}$ in order to solve the initial value problem

$$\text{IVP} \begin{cases} y' = y^2 \\ y(1) = 2. \end{cases}$$

if $y(x) = \frac{1}{C-x}$

$y(1) = \frac{1}{C-1} \overset{\text{want}}{=} 2 \Rightarrow C = 1.5$

$\left(\frac{1}{C-1} = \frac{1}{.5} = 2 \checkmark \right)$

soltn to IVP is

$y(x) = \frac{1}{1.5 - x}$

$$\begin{cases} y' = y^2 \\ y(1) = 2 \end{cases}$$

2c) What is the largest interval on which your solution to (b) is defined as a differentiable function? Why?

$y(1)=2$

$(-\infty, 1.5)$

$-\infty < x < 1.5$

2d) Do you expect that there are any other solutions to the IVP in 2b? Hint: The graph of the IVP solution function we found is superimposed onto a "slope field" below: The line segments at points (x, y) have values y^2, because solutions graphs to the differential equation

$$y' = y^2$$

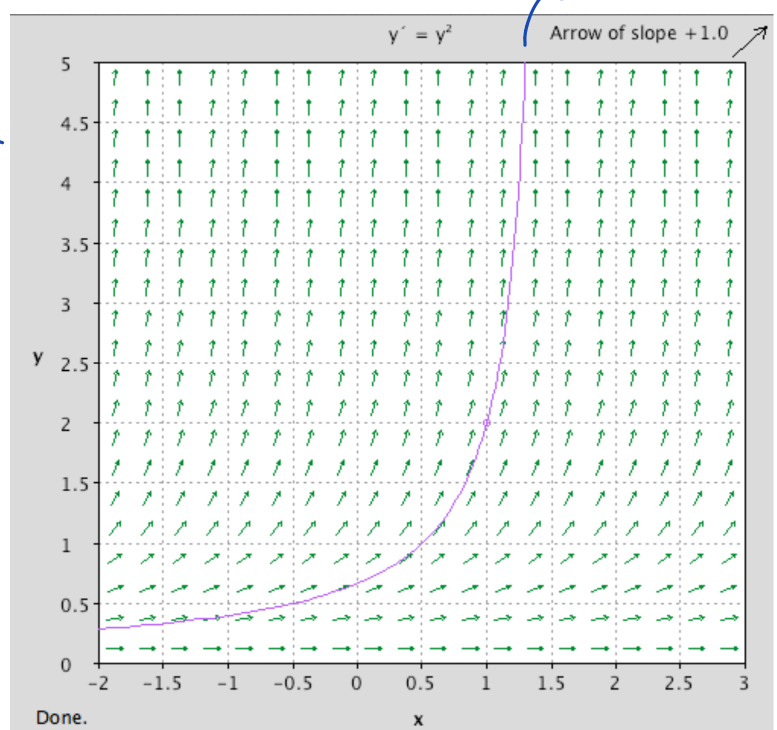
will have slopes given by the derivatives of the solutions $y(x)$. This might give you some intuition about whether you expect more than one solution to the IVP.

$y = \frac{1}{1.5-x}$

$y' = y^2$

at any point on soln graph (x, y)

the slope of graph (y') is y^2



HW text

1.1 : 4, 6, 15, 19