

Math 2250-004

Week 9 concepts and homework, due March 14 at the start of class

Recall that all listed problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Wednesday quiz will be drawn from all of these concepts and from these or related problems.

5.3: using the algorithm for finding the general solution to constant coefficient linear homogeneous differential equations: real roots, Euler's formula and complex roots, repeated roots; solving associated initial value problems.

5.3: 3, 9, 11, 23, 27. (You have a lot of problems like this on your lab.)

w9.1) Do the following problems for homogeneous linear differential equations by hand. They are testing your ability to use the algorithm for finding bases for the solution spaces, based on the characteristic polynomial. Check your work with Wolfram alpha or other software. Hand in a printout or screen shot of your computer verifications for the differential equation solutions, along with your written work. (You have more problems like this on your lab.)

a) Find the general solution to the differential equation for $y(x)$

$$y^{(4)} - 8y' = 0.$$

b) Find the general solution to the differential equation for $y(x)$

$$y^{(5)} + 6y^{(3)} + 9y' = 0.$$

w9.2) Euler's formula

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

is extremely useful in higher mathematics/science/engineering. In class we discuss how this definition is motivated by Taylor series. Amazingly, the rule of exponents is true for such expressions. In other words

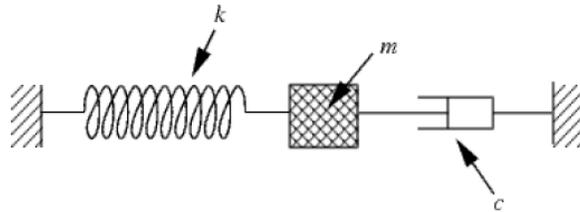
$$e^{i\alpha + i\beta} = e^{i\alpha} e^{i\beta}.$$

Check this identity by rewriting the left side as $e^{i(\alpha + \beta)}$ and then using Euler's formula to expand both sides. You'll notice that the identity is true because of the addition angle formulas for $\cos(\alpha + \beta)$ and $\sin(\alpha + \beta)$. (And so this gives a good way to recover the trig. identities if you happen to forget them.) Magic!

5.4: mechanical vibrations: setting up and solving differential equations and initial value problems for mechanical vibrations; underdamped, critically damped, and overdamped vibration problems; applications to pendulums, mass-spring configurations, and related problems; understanding and using phase-amplitude form for sinusoidal functions.

5.4: 3, 4, 5, 6, 15, 17, 21. (In 15, 17, 21 solve the IVP's only, and indicate whether the configuration is over-damped, under-damped, or critically-damped. Do not graph.)

w9.3 Consider the unforced mass-spring-dashpot system with $m = 3$ and $k = 48$. In the following problems the damping constant c will vary.



So, the displacement $x(t)$ of the mass from its equilibrium position will satisfy the differential equation

$$3x''(t) + cx'(t) + 48x(t) = 0.$$

In each part below, we will consider the IVP with

$$\begin{aligned} x(0) &= 3 \\ x'(0) &= -4. \end{aligned}$$

warning: there will be decimals instead of whole numbers in some parts of this problem - use your calculators or Wolfram alpha as needed for the computations. (And even to check your answers, since Wolfram alpha is happy to solve differential equations and initial value problems.)

a) Solve the IVP when there is no damping, i.e. $c = 0$. After finding your solution in linear combination form, convert into amplitude-phase form. Identify numerical values for amplitude, angular frequency, frequency, period, phase angle, and time delay.

The following parts are now optional - you will have a lab problem this week like this.

b) Solve the IVP when $c = 1.2$ (we refer to the situation when c is small enough so that the characteristic polynomial has complex roots as *underdamped*). Instructions: When finding roots of characteristic polynomials, round off to 5 digits.

c) Recall that a value of the damping constant induces so-called *critical damping* whenever the associated characteristic polynomial has a double real root. Find the value of c that leads to critical damping in this problem, and solve the IVP in this case.

d) Finally, solve the IVP when $c = 51$ (here, we are in the situation of *over-damping*).

e) Use software (Matlab, Wolfram alpha, Maple, etc) to create a display containing the graphs of all four solutions above, on the interval $0 \leq t \leq 4$. Print out a copy, and label which graph corresponds to which solution.