Math 2250-004
Week 7 concepts and homework, due February 28

Recall that all problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Wednesday quiz will be drawn from all of these concepts and from these or related problems.

4.1: *linear combinations of vectors in* $\mathbb{R}^2$ *and* $\mathbb{R}^3$*; linear dependendence and independence; subspaces of* $\mathbb{R}^3$ *;*

w7.1) (fits in 4.1 and 4.3) Consider the three vectors

$$\underline{\boldsymbol{u}} := \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \ \underline{\boldsymbol{v}} := \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}, \ \underline{\boldsymbol{w}} := \begin{bmatrix} -5 \\ 5 \\ 5 \end{bmatrix}.$$

a) Use a reduced row echelon computation to check that the span of these three vectors is not $\mathbb{R}^3$.
b) Use your reduced row echelon form computation to write $\underline{\boldsymbol{w}}$ as a linear combination of $\underline{\boldsymbol{u}}$, $\underline{\boldsymbol{v}}$. (Hint: what augmented matrix would you have if you were solving $c_1\underline{\boldsymbol{u}} + c_2\underline{\boldsymbol{v}} = \underline{\boldsymbol{w}}$ for c_1, c_2 ?)
c) The span of these three vectors is actually just the span of the first two, i.e. a plane through the origin. Find the implicit equation $a\,x + b\,y + c\,z = 0$ satisfied all points (x, y, z)whose position vectors $[x, y, z]^T$ are in the span of $\underline{\boldsymbol{u}}$.$\underline{\boldsymbol{v}}$. For reference in this problem see Exercise 3d in our Wednesday February 21 class notes, which we will finish on Friday February 23 (and which will be posted on our lecture page as feb23. pdf). You are looking for the condition on the vector $\underline{\boldsymbol{b}}$ so that it is a linear combination of $\underline{\boldsymbol{u}}$, $\underline{\boldsymbol{v}}$, i.e. trying to see for which $\underline{\boldsymbol{b}}$ the system

$$c_1\underline{\boldsymbol{u}} + c_2\underline{\boldsymbol{v}} = \underline{\boldsymbol{b}}$$

has solutions $\underline{\boldsymbol{c}}$.

4.2: *subspaces of* $\mathbb{R}^n$ *, expressed either as the span of a collection of vectors and/or as the solution space of a homogeneous matrix equation* $A\underline{\mathbf{x}} = \underline{\mathbf{0}}$ *.*
3,**4**,5, **6**, **9**, **11**, **12**, 15, 17, 19, 24, **27**, **29**; (By the way, problem 29 is the general fact connected to what you observe in your lab 6 problem 3d.)

w7.2) Consider the homogenous matrix equation

$$\begin{bmatrix} 1 & 3 & 0 & 5 \\ 1 & -2 & 5 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Express the solution space of this matrix equation (which is a subspace of $\mathbb{R}^4$) as the span of two vectors. Hint: Use Chapter 3 techniques.

4.3: *testing for independence of vectors; using reduced row echelon form to find dependencies for vectors in the columns of a matrix.*
1, 3, 6, **8**, 9, **10**, 16, 17, 18, **23**, 25. (See also **w7.1** above.)

4.4: *bases for subspaces:* 1, **2**, 3, **4** ,**6**, 15.

w7.3a Use a determinant computation to verify that

$$\left\{ \underline{u} := \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \ \underline{v} := \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}, \ \underline{w} := \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$$

is a basis for $\mathbb{R}^3$.

b) Why are the three vectors in problem **w7.1** not a basis for $\mathbb{R}^3$?