Math 2250-004

Week 6 -7 concepts and homework, due Thursday Feb 22 at the start of your lab

Recall that all problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

3.6 Determinants

Cofactor expansions: 3, **6.**

Combining cofactor expansions with elementary row operations to compute determinants: 11, 17.

The adjoint formula for matrix inverses 25, 33, *and Cramer's rule for finding individual components of the solution vector:* 21, 31.

w6.1a) Use Cramer's rule to re-solve for x and y in the linear system **w5.1c.**

w6.1b) Compute the determinants of the two matrices in **w5.2**, and verify that the determinant test correctly identifies the invertible matrix.

w6.1c) Use the adjoint formula to re-find B^{-1} in **w5.2**.

w6.1d) Use B^{-1} to solve the system

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 2 \\ -2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

w6.3e) Re-solve for the y -variable in **w6.1d)**, using Cramer's Rule.

From homework 5, these are the problems refered to in the **w6.1** problem above:

w5.1c) Use your formula for A^{-1} to solve the system

$$\begin{bmatrix} 5 & 2 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}.$$

w5.2) Use the Gaussian elimination algorithm to determine that the matrix A below is not invertible, whereas the matrix B is. Use the algorithm that begins by augmenting a matrix with the identity matrix, in order to find the inverse matrix B^{-1}.

$$A := \begin{bmatrix} -1 & -4 & 1 \\ -1 & 2 & -1 \\ 4 & 1 & 1 \end{bmatrix} \qquad B := \begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 2 \\ -2 & -2 & 1 \end{bmatrix}$$

4.1: linear combinations of vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$; linear dependendence and independence; subspaces of $\mathbb{R}^3$;
1, 3, 7, 9, **11**, 15, 16, 22, 25, **26**, 33.

w6.2) Consider the three vectors

$$\underline{\boldsymbol{u}} := \begin{bmatrix} 3 \\ -2 \end{bmatrix} \quad \underline{\boldsymbol{v}} := \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \underline{\boldsymbol{w}} := \begin{bmatrix} 9 \\ 2 \end{bmatrix}$$

a) Compute the magnitudes (lengths), $|\underline{\boldsymbol{u}}|$ and $|\underline{\boldsymbol{v}}|$.
b) Express $\underline{\boldsymbol{w}}$ as a linear combination of $\underline{\boldsymbol{u}}$ and $\underline{\boldsymbol{v}}$, by solving the appropriate linear system.
c) Make a careful and accurate sketch which illustrates your answer to (b), as we will do in Exercise 1 of the Wednesday February 21 class notes. (You can print off free graph paper at http://www.printfreegraphpaper.com/)
d) Find a linear combination of $\underline{\boldsymbol{u}}$, $\underline{\boldsymbol{v}}$, $\underline{\boldsymbol{w}}$ which adds up to the zero vector. (You've already done the work for this in part (c), if you just rearrange your equation!) Illustrate this linear combination adding up to zero on your sketch for (c).