Math 2250-004
Week 4 concepts and homework, due February 7

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Wednesday quiz will be drawn from all of these concepts and from these or related problems.

3.1-3.3: linear systems of (algebraic) equations; augmented matrices and Gaussian elimination to compute reduced row-echelon form. Explicitly specifying the solution space by backsolving the reduced row echelon form of the augmented matrix. Depending on your previous experience and background, you may need to practice this algebra a lot, or just a little. So, I've included a fairly large number of good practice problems in addition to the ones you'll hand in.

3.1: 1, **4**, **6**, 11, **16**, 17, 19, 23, 24, 27, **28**, 29, 32, 33, **34**
3.2: 7, **8**, 9, 13, 17, **20** (backsolve from row echelon form), 29, 30;
3.3: 13, 17, **20**, 33, **34**;

w4.1 Find the equation of the parabolic graph that passes through the points $(1,0)$, $(2,3)$, $(4,3)$. Exhibit and solve they system of three equations in the three unknowns a, b, c for the unknown parabola $y = ax^2 + bx + c$. Use Gaussian elimination to compute the reduced row echelon form of the augmented matrix, in order to find a, b, c. Sketch the resulting parabola by hand, together with the three points it passes through.

w4.2 Consider the augmented matrix for 3.2.20 above:

$$\left[\begin{array}{ccccc|c} 2 & 4 & -1 & -2 & 2 & 6 \\ 1 & 3 & 2 & -7 & 3 & 9 \\ 5 & 8 & -7 & 6 & 1 & 4 \end{array}\right].$$

Continue your work from **3.2.20** to compute the reduced row echelon form of this augmented matrix, and verify that you recover the same explicit solutions as you did in that problem, when you backsolved from a row echelon form.

w4.3) Runge-Kutta is based on Simpson's rule for numerical integration. Simpson's rule is based on the fact that for a subinterval $[d, d+h]$ of length h, the parabola $y = p(x)$ which passes through the points (d, y_0), $\left(d + \frac{h}{2}, y_1\right)$, $(d + h, y_2)$ has integral

$$\int_d^{d+h} p(x)\, dx = \frac{h}{6} \cdot (y_0 + 4y_1 + y_2).$$

w4.3a) The integral approximation above follows from one on the interval $[-1, 1]$ by an affine change of variables. So first consider the interval $[-1, 1]$. We wish to find the parabolic function

$$q(x) = a\,x^2 + b\,x + c$$

with unknown parameters a, b, c. We want $q(-1) = y_0$, $q(0) = y_1$, $q(1) = y_2$. This gives 3 equations in 3 unknowns, to find a, b, c in terms of y_0, y_1, y_2. Write down these linear equations and find a, b, c.

w4.3b) Compute

$\int_{-1}^{1} q(x)\, dx$ for thes values of a, b, c you find in part a, and verify the identity

$$\int_{-1}^{1} q(x)\, dx = \frac{1}{3}\left(y_0 + 4y_1 + y_2\right)$$

Note, the formula for general interval follows from a change of variables, as indicated below: