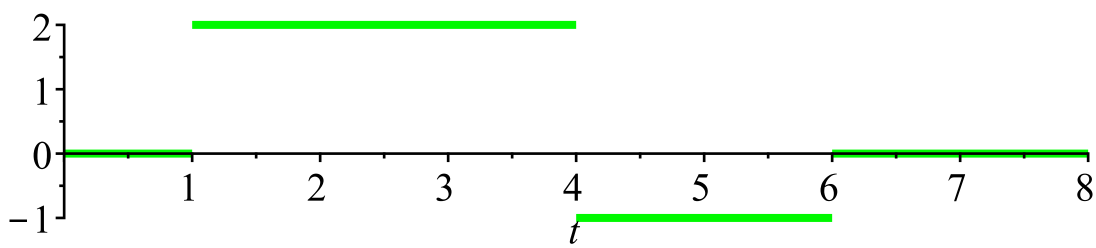
Math 2250-004

Week 12-13 concepts and homework, due Wednesday April 11 at the start of class

Recall that all problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Wednesday quiz will be drawn from all of these concepts and from these or related problems.

10.5: *Piecewise continuous input functions:*
10.5: 3, 7, 11, 13, **31**, 32

w12.1 *Consider the function $f(t)$ which is zero for $t > 6$, is piecewise constant, and has this graph*



Last week you found the Laplace transform $F(s)$ of $f(t)$ directly from the integral definition of Laplace transform. This week, rewrite $f(t)$ as a linear combination of translated unit step functions and compute its Laplace transform using the table entry

$$\mathcal{L}\{u(t-a)\}(s) = \frac{e^{-a s}}{s}.$$

10.4: using convolution integrals to find inverse Laplace of F(s)G(s); using the differentiation theorem to find the Laplace transform of $t \cdot f(t)$; using these (and other) techniques to solve linear differential equations.

w12.2a) Compute the convolution $f*g(t)$ for $f(t) = t$ and $g(t) = t^2$ and use the Laplace transform table to show that $\mathcal{L}\{f*g(t)\}(s) = F(s)G(s)$ in this case. Be clever about which order you write the f, g terms in the convolution integral, as one way is easier than the other.

b) Repeat part a for the example $f(t) = t$, $g(t) = \sin(k\,t)$.

10.4: 2, 3, 9, **36**, **37**, 38. Note: These two problems are very quick applications of the convolution table entry for Laplace transforms.

w12.3) Redo 10.5.31 using the convolution table entry to find the solution $x(t)$ to the initial value problem:

$$x''(t) + 4\,x(t) = f(t)$$
$$x(0) = 0$$
$$x'(0) = 0$$

where the forcing function is given by

$$f(t) = \begin{cases} 1 & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases}.$$

EP 7.6 Engineering applications: Duhamel's Principle and delta function forcing.
EP 7.6 1, **2**

6.1: finding eigenvalues and eigenvectors (or a basis for the eigenspace if the eigenspace is more than one-dimensional).
6.1: 7, 17, 19, 25
w12.4) Find eigenvalues and eigenvectors (a basis for the eigenspace), for the following matrices. You can check your work with technology, but you don't have to hand in the technology check.

a) $A := \begin{bmatrix} -1 & -2 \\ 4 & 5 \end{bmatrix}$

b) $B := \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$

c) $C := \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.