Math 2250-004

Week 11-12 concepts and homework, due Thursday April 5 at the start of your labs

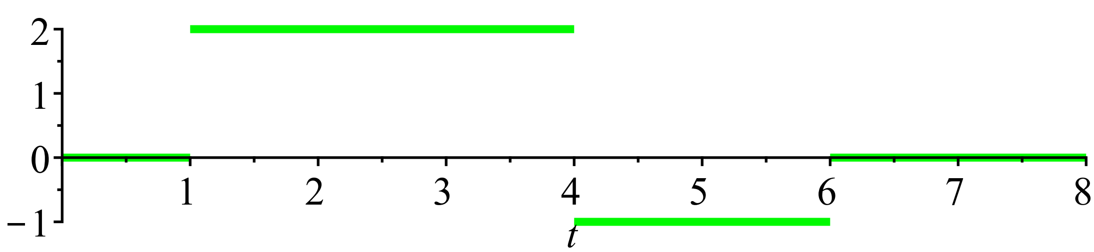
Recall that all problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in. When you are asked to check answers with technology, please hand in a screen shot or output from the check. You can do all of the technology checks together and put them at the end of your assignment. Make sure to indicate clearly which checks correspond to which problems.

10.1: Laplace transforms and inverse transforms.
Use the definition of Laplace transform and integration techniques to compute Laplace transforms of $f(t)$*:*
1, 3, 7, 9
w12.1) Use the definition of Laplace transform

$$\mathcal{L}\{f(t)\}(s) = \int_0^\infty f(t)e^{-st}\,dt$$

and integration by parts to compute the Laplace transform $F(s)$ of $f(t) = t\,e^{2t}$.

w12.2) Consider the function $f(t)$ which is zero for $t > 6$, is piecewise constant, and has this graph



Break the Laplace transform integral from zero to infinity into the sum of integrals over four subintervals (two of the integrals will be zero and use antidifferentiation for the other two), in order to compute the Laplace transform $F(s)$ of $f(t)$.

Use Laplace transform table, linearity (and useful trig identities) to compute Laplace transforms and inverse Laplace transforms.
13, 17, 19, 23, 29
w12.3a) Use the Laplace transform table in the front cover of the text, algebra, and the linearity of Laplace transform, to compute the Laplace transform $F(s)$ for

$$f(t) = 2\,e^{-t}\sin(3\,t) + 4\,e^{-3\,t}(1 - 4\,t) - 2\,t^2.$$

b) Check your answer with technology. (Wolfram alpha examples for Laplace transform, inverse Laplace transform, and partial fractions are shown at the end of this document.)

w12.4a) Use the Laplace transform table and linearity to compute the inverse Laplace transform $g(t)$ of

$$G(s) = \frac{3}{s+2} + \frac{1}{(s-3)^2} - \frac{3\,s+6}{s^2+9} + \frac{2\,s+3}{\left(s^2+4\right)^2}.$$

b) Check your answer with technology.

10.2: Transforming and solving initial value problems via Laplace transforms;
3, 7, 9,19.

w11.5) Practice completing the square (and "completing the linear" in the numerator) by finding the inverse Laplace transform $f(t)$ for $F(s) = \dfrac{4\,s + 7}{s^2 + 2\,s + 5}$.

w11.6) Use Laplace transforms to solve the underdamped initial value problem

$$x''(t) + 4\,x'(t) + 13\,x(t) = 0$$
$$x(0) = 1$$
$$x'(0) = 4$$

w11.7a) Use Laplace transforms (and partial fractions) to solve the initial value problem for $x(t)$:

$$x''(t) + 6\,x'(t) + 9\,x(t) = 30\cos(3\,t)$$
$$x(0) = 0$$
$$x'(0) = 0$$

b) Check your final answer with technology.
c) Identify the steady periodic solution, and convert it to amplitude-phase form.

10.2-10.3: Laplace transform table entries; partial fractions to simplify F(s); the translation theorem with completing the square, to identify inverse Laplace transforms; applying these and other techniques to initial value problems.
10.2: **20**
10.3: 3, 7, 9, 17, **20**, 27, **30**, **32**, 34.

w11.8) With access to a Laplace transform table it is possible to very quickly recover the general solutions to key mechanical oscillation problems (some of which are real messy to derive with Chapter 5 techniques) . Do this for

a) undamped forced oscillation, $\omega \neq \omega_0$:

$$x''(t) + \omega_0^2\,x(t) = \frac{F_0}{m}\cos(\omega\,t)$$
$$x(0) = x_0$$
$$x'(0) = v_0$$

b) undamped forced oscillation, $\omega = \omega_0$:

$$x''(t) + \omega_0^2\,x(t) = \frac{F_0}{m}\cos(\omega_0\,t)$$
$$x(0) = x_0$$
$$x'(0) = v_0$$

Notes: Wolfram alpha can check all your steps in Laplace transform and inverse Laplace transform problems...

laplace transform

Web Apps | Examples | Random

Assuming "laplace transform" refers to a computation | Use as referring to a mathematical definition or a general topic or a function instead

- function to transform: t*cos(3*t)
- initial variable: t
- transform variable: s

Input:

$\mathcal{L}_t[t\cos(3t)](s)$

Open code

$\mathcal{L}_t[f(t)](s)$ is the Laplace transform of $f(t)$ with complex argument s

Result:

$$\frac{s^2-9}{(s^2+9)^2}$$

WolframAlpha computational knowledge engine.

inverse laplace transform

Web Apps | Examples | Random

Assuming "inverse laplace transform" refers to a computation | Use as referring to a mathematical definition instead

- function to transform: (s^2-9)/(s^2+9)^2
- initial variable: s
- transform variable: t

Input:

$$\mathcal{L}_s^{-1}\left[\frac{s^2-9}{(s^2+9)^2}\right](t)$$

Open code

$\mathcal{L}_s^{-1}[f(s)](t)$ is the inverse Laplace transform of $f(s)$ with real variable t

Result:

$t\cos(3t)$

WolframAlpha computational knowledge engine.

partial fractions

Web Apps | Examples | Random

Assuming "partial fractions" refers to a computation | Use as a general topic instead

- rational function: (s^2-9)/(s^2+9)^2

Input:

partial fractions	$\frac{s^2-9}{(s^2+9)^2}$

Open code

Result:

Step-by-step solution

$$\frac{s^2-9}{(s^2+9)^2} = \frac{1}{s^2+9} - \frac{18}{(s^2+9)^2}$$